VANIER COLLEGE, DEPARTMENT OF MATHEMATICS
LINEAR ALGEBRA, H18, TEST 2

NAME: _____ STUDENT ID: _____

(1) (2.5 marks) a) Compute the determinant of

$$B = \begin{pmatrix} 3 & 3 & 0 & 5 \\ 2 & 2 & 0 & -2 \\ 4 & 1 & -3 & 0 \\ 2 & 10 & 3 & 2 \end{pmatrix}$$

b) If a square matrix X obeys the equation

$$B^T X^{-1} B^{-1} = 2XB^{-2}X^{-1},$$

where B is the matrix above, use properties of determinants to compute $detX$.

(2) (2.5 marks) Compute the adjugate and the inverse of the matrix

$$A = \begin{pmatrix} 1 & -1 & 0 \\ -1 & 1 & 1 \\ 0 & -1 & 1 \end{pmatrix}$$

b) Solve the linear system

$$\begin{aligned} x - y &= 2 \\ -x + y + z &= -2 \\ -y + z &= -3 \end{aligned}$$

using Cramers Rule.

(3) (2.5 marks) Consider the two lines

$$L_1 = \begin{cases} x = 5 - 2t \\ y = 2 + 2t \\ z = -1 - 4t \end{cases}, \qquad L_2 = \begin{cases} x = 1 + s \\ y = -2 - s \\ z = -3 + 2s \end{cases}$$

a) Determine the shortest distance between L_1 and L_2.

b) Determine the equation of the plane containing both lines.

(4) (2.5 marks) Consider the two planes

$$P_1 : \ 3x - y - z = 5 \quad \text{and} \quad P_2 : \ 7y - 9z = 4$$

a) Find the angle between these two planes.

b) Find the equations of the line L of intersection of these two planes.

c) Find the equations of the line L_1 through the point $(-1, -1, 2)^T$ and parallel to both planes.

(5) (2.5 marks) a) Conisider the line $L = \{x = 1 - u, y = -3 + 2u, z = 1 - 2u\}$ and the planes $P_1 : x - 3y + z = 2$ and $P_2 : x - 3y + z = 20$.

a) Determine the points of intersection of the line L with the planes P_1 and P_2.

b) Determine the midpoint, M, of the interval connecting the planes P_1 and P_2 along the line L.

c) Determine the shortest distance between midpoint M and the plane P_1.

(6) (2.5 marks) Data has been accumulated on the heights of children relative to their parents. Suppose that the probabilities that a tall parent will have a tall, medium-height or short child are 0.6, 0.2 and 0.2 respectively; the probabilities that a medium height parrent will have a tall, medium-height or short child are 0.1, 0.7 and 0.2 respectively; and the probabilities that a short parent will have a tall, medium-height, or short child are 0.2, 0.4 and 0.4, respectively.
a) Write down the transition matrix for this Markov chain.

b) What is the probability that a short person will have a tall grandchild?

c) Find the proportions of the population consisting of tall, medium-height and short people?

(7) (2.5 marks) Consider the points $A = (-1, -1, 0)^T, B = (-1, 1, 3)^T, C = (0, 3, 8)^T$ and $D = (2, 4, -1)^T$ in $\mathbb{R}^3$.

a) Find the volume of the parallelepiped PRL determined by the points A, B, C, D.

b) Find the area of the face of PRL determined by the points A, C, D.

c) Find the shortest distance between the vertex B and the face determined by the points A, C, D.

(8) (2.5 marks) Determine if the two lines $L_1 = \{x = 1 + 4t, y = 4t, z = 7 - 2t\}$ and $L_2 = \{x = 3 + 2s, y = 10 + 6s, z = 6 - s\}$ intersect and if they do determine the line orthogonal to both L_1 and L_2 and passing through the point of intersection of L_1 and L_2.