Name:__________________________ October 16, 2020

CALCULUS 1 - TEST 1

(1) (*16 marks*)

Compute the limits

(a) $\lim_{x\to 2} \frac{2x^4 - 32}{x-2}$ (b) $\lim_{x\to 0} \frac{|3x-1| - |3x+1|}{x}$ (c) $\lim_{x\to 0} x\left(1 + \sin\left(\frac{1}{x^2}\right)\right)$

(2) (*14 marks*)

Compute the limits

(a) $\lim_{x\to -\infty} \frac{x}{\sqrt{x^2+1}}$ (b) $\lim_{x\to \infty} (\sqrt{x^2 - x - 10} - x)$

(3) (*12 marks*)

Consider the function

$$f(x) = \frac{kx\sqrt{1-x^2}}{x-1}$$

where k is a positive constant (i.e. $k > 0$).

Evaluate each of the following limits. If the limit does not exist, state why.

(a) $\lim_{x\to -1^+} f(x)$ (b) $\lim_{x\to +1^-} f(x)$

(4) (*12 marks*)

Determine the values of L and M so that $\lim_{x\to\infty}\left(\frac{x^2+1}{x+1} - Lx - M\right) = 3$

(5) (*8 marks*)

Suppose that $f(x)$ is an odd function that is right continuous at 0, (i.e. $\lim_{x\to 0^+} f(x) = f(0)$). Using algebraic arguments show that $f(x)$ is continuous at 0.

(6) (*10 marks*)

Suppose that f is a continuous function on the closed interval $[0,1]$ and that $0 \le f(x) \le 1$ for every x in $[0,1]$. Prove that there exists a number $c \in [0,1]$ such that $f(c) = c$.
[**Hint:** Apply the IVT to $g(x) = f(x) - x$.]

(7) (*14 marks*)

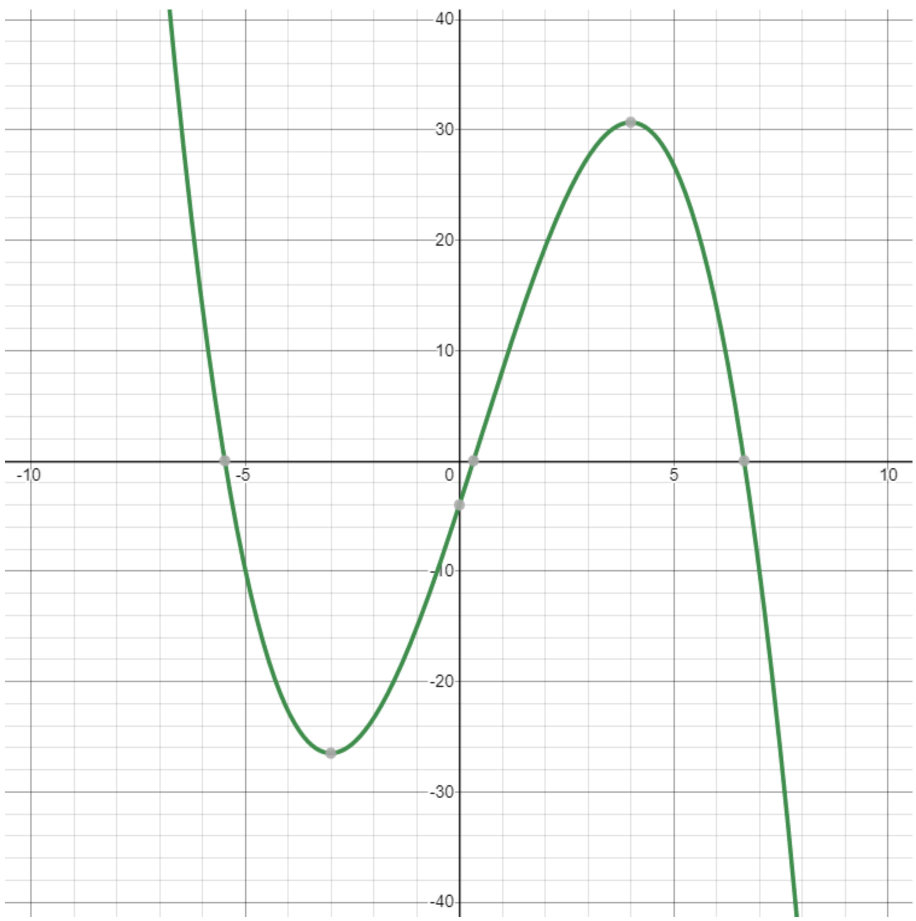
Let $k(x) = \dfrac{x}{x^2+1}$

(a) Use the definition of the derivative (i.e. Newton's quotient) to compute the $k'(x)$

(b) Determine the equation of the tangent line to the graph of $y = \dfrac{x}{x^2+1}$ at $\left(1, \frac{1}{2}\right)$

(8) (*14 marks*)

Consider the function $f(x)$ whose graph is drawn below.



(a) Draw the graphs of $f'(x)$ and $f''(x)$ under the graph of $f(x)$. Clearly label each graph.

(b) For what values of x does $f(x)$ have a horizontal tangent line? State the interval(s) on which $f(x)$ is increasing on. State the interval(s) for which $f(x)$ is decreasing on. What is the behaviour of $f'(x)$ on these intervals?

(c) State the interval(s) on which $f'(x)$ is increasing on. State the interval(s) for which $f'(x)$ is decreasing on. What is the behaviour of $f''(x)$ on these intervals?