DIFFERENTIAL EQUATIONS, H19, TEST 1

(1) (2 marks) Find the solution $y(t)$ of the initial value problem

$$ty' + 2y = \sin 2t, \quad y(\pi/2) = -1.$$

On what interval is the solution you found defined?

(2) (2.5 marks) A chemical reaction involves the interaction of one molecule of substance P with one molecule of substance Q to produce one molecule of new substance X. Suppose that p and q are the initial abundances (in moles) of P and Q respectively. Let $x(t)$ be the amount of X produced by the reaction at time t (in minutes). The rate at which the reaction occurs is given by the equation

$$\frac{dx}{dt} = 0.4(p - x)(q - x).$$

If $p = 6$ and $q = 8$ determine the amount of X as a function of time, i.e. solve the DE for the function $x(t)$. How long it will take for all of P to be completely consumed by the reaction; argue by taking a limit $t \to \infty$ of an appropriate quantity.

(3) (2.5 marks) Find a solution for the initial value problem

$$ye^{2xy} + x + xe^{2xy}y' = 0, \quad y(1) = 0.$$

What is the interval of existence of the solution?

(4) (2.5 marks) Use Euler's method with stepsize $h = 0.2$ to approximate the solution of the initial value problem

$$y' = (y+1)^2, \quad y(0) = 3$$

on the interval $0 \le t \le 1$. Organize your calculations in a table.

Now compute the exact solution and the interval on which it is defined. Are the Euler approximations you just computed meaningful? Explain.

(5) (2 marks) Determine the solution of the initial value problem

$$9y'' + 6y' + y = 0, \quad y(0) = -3,\ y'(0) = 1$$

and describe the behaviour of the solution as t increases.

(6) (2.5 marks) Determine the solution of the initial value problem

$$y'' + 2y' + 1.25y = 0, \quad y(0) = 3,\ y'(0) = 2,$$

sketch the graph of the solution and describe its behaviour as t increases.

(7) (3 marks) Use the method of undetermined coefficients to solve the initial value problem

$$y'' + 2y' = \frac{3}{2}te^{-2t}, \quad y(0) = 1,\ y'(0) = 1.$$

(8) (3 marks) Verify that the functions $y_1(t) = t$ and $y_2(t) = t \ln t$ are solutions of the homogeneous equation $t^2y'' - ty' + y = 0$. Use these results and the method of variation of parameters to solve the initial value problem

$$t^2y'' - ty' + y = t, \quad y(1) = 1,\ y'(1) = -4.$$