CALCULUS II, H21, FINAL EXAMINATION

Name: ________________________ Student number______________________

(1) (10 marks) Determine whether the following series are convergent or divergent. State clearly which test you are using.

$$a) \quad \sum_{n=1}^{\infty} \frac{\arctan n}{n^3 + 1}$$

$$b) \quad \sum_{n=1}^{\infty} \frac{e^{1/n}}{n^2}$$

$$c)\quad \sum_{n=0}^{\infty} \frac{\sqrt{n}}{n+4}$$

$$d)\quad \sum_{n=1}^{\infty} \frac{(2n)!}{n^n}$$

$$e)\quad \sum_{n=1}^{\infty} \frac{2^{n+2}}{(\ln n+1)^n}$$

(2) (4 marks) Determine the sum of the series

$$\sum_{n=1}^{\infty} \frac{(-3)^{n+1}}{2^{(2n-1)}}$$

(3) (5 marks) Determine if the series

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln n}$$

is conditionally convergent, absolutely convergent or divergent.

(4) (6 marks) Determine the interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{(n+5)12^n}.$$

(5) (6 marks) The Maclaurin series for e^x is $\sum_{n=0}^{\infty} x^n/n!$ Use this result to find the Maclaurin series for

$$I(x) = \int_0^x e^{-\frac{t^2}{2}}\, dt.$$

Next evaluate $I(1)$ with error less than 10^{-6}.

(6) (5 marks) Use the definition of the Taylor series to find the first four nonzero terms of the series for $f(x) = x^{2/3}$ centered at $x = 1$. Next use this result to find the first three nonzero terms in the Taylor series for $f'(x)$ centered at $x = 1$.

(7) (5 marks) Consider the integral $\int_0^6 (5x^2 - 3x)\ dx$.

a) Use right endpoints and n subintervals to find a formula for the Riemann sum.

b) Evaluate the integral by finding the limit of the Riemann sum as $n \to \infty$.

Here are three useful sums:

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}, \quad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}, \quad \sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}.$$

(8) (3 marks) Determine the derivative with respect to x of the following function

$$f(x) = \int_1^{\sqrt{x}} \frac{t^2 - 10}{\ln t + 2}\, dt.$$

Is $f(9)$ a postive or a negative number. Explain your answer.

(9) (5 marks) Determine the area under the graph of $f(x) = 1/(2 + e^{-x})$ and above the x-axis interval $[0, 4]$.

(10) (5 marks) Evaluate the following integrals

$$a) \quad \int x^3 \ln x \; dx$$

$$b) \quad \int_0^{\pi/4} \tan^3 x \sec^3 x \; dx.$$

(11) (7 marks) Find the exact length of the curve $y = \ln(1 - x^2), 0 \leq x \leq 1/2$.

(12) (3 marks) The integral below describes the volume of a solid.

$$V = \pi \int_1^4 \left[9 - (3 - \sqrt{x})^2\right] \, dx$$

Describe the solid. Make sure to include a drawing. You do not have to evaluate the integral.

(13) (4 marks) Set-up an integral which uses the method of cylindrical shells to find the volume of the solid generated when the region in the first quadrant bounded by the curves $y = 2x^2$ and $y = x^2 + 1$ is rotated about the axis $x = 2$. You do not have to evaluate the integral, but make sure to include a drawing.

(14) (6 marks) Investigate the improper integrals

$$a)\quad \int_{-\infty}^{1} ze^{2z}\ dz$$

$$b)\quad \int_{0}^{\pi/3} \frac{\cos\theta}{\sqrt{\sin\theta}}\ dx$$

(15) (6 marks) Helped by global warming Eastern Cottontail rabbits have crossed the St. Laurence river and are colonizing a village on the North shore. The population expansion is well modelled by the differential equation

$$\frac{dP}{dt} = 0.1P\left(1 - \frac{P}{1000}\right)$$

where time is months. If initially (May, 2021) there were 40 cottontails in the village area, determine there popualtion as a function of time, $P(t)$. How long it will take for the population to reach 800 individuals?