# PROBABILITY AND STATISTICS, A22, TEST 1

Name: _____                    Student number_____

(1) (2.5 marks) i) Let $A$ and $B$ be mutually exclusive events with $p(A) = 0.5$ and $p(B) = 0.3$. Determine $p(A' \cap B')$ and also $p(A|B)$.

   ii) Let $C$ and $D$ be independent events with $p(C) = 0.5$ and $p(D) = 0.3$. Determine $p(C' \cup D)$.

(2) (2.5 marks) Diseases I and II are prevalent among people in a certain population. It is known that in their lifetime, 8% of the population will contract disease I, 15% will contract disease II, and 2% will contract both diseases. Determine the probability that a randomly chosen person from this population will not contract both diseases, given that he or she has contracted at least one disease.

(3) (2.5 marks) An insurance company offers its policy holders a number of different premium payment options. For a randomly selected policyholder, let X be the number of months between successive payments. The cumulative distribution function of $X$ is as follows

$$F(x) = \begin{cases} 0 & x < 1 \\ 0.4 & 1 \leq x < 3 \\ 0.5 & 3 \leq x < 6 \\ 0.8 & 6 \leq x < 12 \\ 1 & 12 \leq x \end{cases}$$

i) Compute $p(2 \leq X \leq 6)$ and $p(X \geq 4)$.

ii Compute the mean and the standard deviation of $X$.

(4) (2.5 marks) Every day, a weather forecaster predicts whether or not it will rain. For 95% of rainy days, she correctly predicts that it will rain. For 90% of non-rainy days, she correctly predicts it will not rain. Suppose that 20% of days are rainy and 80% are non-rainy.

i) If this forecaster predicts tomorrow will be a rainy day, what is the probability that it will actually rain?

ii) What is the probability that the forecaster will be correct seven days in a row?

(5) (2.5 marks) In a week in September 2022 an art dealer received 7 pictures of paintings she might want to acquire for her gallery. The dealer estimates that currently 55% of all submissions are generated by an AI.

i) What is the expected number and the standard deviation of the number of submissions generated by an AI out of the 7?

ii) What is the probability that at least 6 of the 7 paintings are generated by an AI?

iii) The dealer receves information that one of the paintings is definitely generated by an AI. What is her estimate for the probability that at least 6 out of the 7 paintings are generated by AI given this information?

(6) (2.5 marks) i) If $A$, $B$ abd $C$ are three nonempty and nondisjoint events prove that

$$\frac{p(A|B \cap C)}{p(A|C)} = \frac{p(B|A \cap C)}{p(B|C)}$$

ii) If moreover the events $A$, $B$ and $C$ are independent prove that the events $A \cup B'$ and $C'$ are independent.

(7) (2.5 marks) A team of three hippos, five rhinos, six elephants and a giraffe enters a tug-of-war tournament. The position of the players on the rope is selected at random.

a) What is the probability that all the rhinos will be next to each other?

b) What is the probability that all the rhinos will be next to each other and all the elephants will be next to each other?

c) What is the probability that all the rhinos will be next to each other and all the elephants will be next to each other if all the hippos are next to each other?

(8) (2.5 marks) Consider the following game. A player rolls a fair die. If she rolls 3 or less she losses immediatelly. Otherwise she selects, at random, as many cards from a full deck as the number that came up on the die. The player wins 20000\$ if there are three Aces among the selected cards, 100000\$ if there are four Aces among the selected cards and losses otherwise.

i) What is a fair entry fee for this game (the expected value of prize money)?

ii) What is the standard deviation of the prize money?