PROBABILITY AND STATISTICS, A22, FINAL EXAM

Name: _____ Student number_____

(1) (3 marks) A survey found that 45% of the residents of the Lanaudiere region support spending tax funds on conservation projects, 42% oppose this and 13% have no opinion on the matter. The Lanaudiere village of St-Volk has 576 adult residents, who will soon be polled on the idea to use tax funds to buy 56 acres of woods on the edge of the village from the current owners and protect the woods from development. Estimate the probability that between 240 and 250 (inclusive) of the adult residents of St-Volk would support bying these woods.

(2) (3 marks) The cumulative distribution function of a continuous random variable X is given by

$$F(x) = \begin{cases} 0 & x < 0, \\ x(2-x)/2 & 0 \leq x < 1, \\ 1 - x + x^2/2 & 1 \leq x < 2, \\ 1 & 2 \leq x \end{cases}$$

a) Determine the value of X which corresponds to the 20th percentile.
b) Determine the expected value of X.
c) Determine the standard deviation of X.

(3) (3 marks) Large flocks of wild turkeys live in the woods of St-Volk. A random sample of 21 turkeys from these flocks collected in December shows an average weight of 4.8 kg with sample standard deviation of 1 kg. Another random sample of 16 turkeys from the same flocks collected in April shows an average weight of 3.2 kg with sample standard deviation of 0.8 kg. Assume the populations normal: $N(\mu_1, \sigma_1)$ in December and $N(\mu_2, \sigma_2)$ in April.

a) Test $H_0 : \sigma_1 = \sigma_2$ versus $H_1 : \sigma_1 > \sigma_2$ at $\alpha = 0.05$ level of significance. Give a range for the p-value. Draw a conclusion in the context of the problem?

b) Test $H_0 : \mu_1 = \mu_2$ versus $H_1 : \mu_1 > \mu_2$ at $\alpha = 0.05$ level of significance. Give a range for the p-value. Draw a conclusion in the context of the problem.

(4) (3 marks) Many of villagers from St-Volk do some of their shopping at the Walmart at the town of Rainville. Amelia is a resident of St-Volk, hourly associate at the Rainville Walmart and a stats student. She has collected extensive data on the number of monthly visits of the St-Volk residents (the 576 adults) at the Walmart and models the distribution as:

X	0	1	2	3	4
$p(x)$	0.12	0.44	0.28	0.14	0.02

Here X is the number of Walmart visits per adult St-Volk resident per month.

a) Compute the mean number of monthly visits per person and the standard deviation.

b) Estimate the probability that there will be more than 10,300 visits by St-Volk customers at the Rainville Walmart in a year.

(5) (3 marks) Let μ denote the true average number of hours the hourly associates at the Walmart work per week. Assume that the population is normal with standard deviation of 2.5 hours.

a) Test $H_0 : \mu = 12.8$ versus $H_1 : \mu > 12.8$ based on a sample of size $n = 27$ with sample mean $\bar{x} = 13.2$ at $\alpha = 0.01$ level of significance. Report a p-value and draw a conclusion in the context of the problem.

b) What is the power of this test to discriminate a true population mean of 13.3 hours from the hypothesized mean?

(6) (3 marks) Curbside-pickup customers show up the Rainville Walmart at the rate of 4.8 per hour.

a) Determine the probability that there will be only one curbside-puckup customer in half an hour.

b) An hour has passed with only two curbside-pickup customers showing up. Determine the probability that more than additional one and a half hours will ellapse before five more curbside-pickup customers will show up.

c) Determine the probability that no customers will show up between 9am and 9.40am.

(7) (3 marks) Nina is a regular customer at the Rainville Walmart. She claims that her visits take less than 20 minutes spent inside the store on average. As a stats student Amelia carely checks quantitative claims of this type. Amelia has collected sample of 6 visits by Nina to the store. The times spent inside are (in minutes):

$$18.2 \quad 26.4 \quad 9.8 \quad 33.8 \quad 12.2 \quad 10.6$$

Construct a 99% confidence interval for mean time Nina spends inside the store. Based on this interval is Nina's claim correct?

(8) (3 marks) Random variable X has the pdf $p(x) = 2/x^2,\ 1 \leq x \leq 2$. Check the 'Theorem of the Unconscious Statistian' on this example. Determine $E(-X^{1/3})$ in two ways:
   a) Without finding the pdf of $Y = -X^{1/3}$.
   b) By first computing the pdf of $Y = -X^{1/3}$.

(9) (3 marks) Amelia has also collected data on her adventurous house cat Erlang which dares to walk through the woods of St-Volk. A sample of 7 such trips show the following durations (in hours):

$$0.8 \quad 2.3 \quad 1.1 \quad 0.7 \quad 1.8 \quad 1.5 \quad 1.6$$

Assume the these times are normally distributed.

a) Test $H_0 : \mu = 1.3$ versus $H_1 : \mu > 1.3$ at $\alpha = 0.01$ level of significance. Draw a conclusion in the context of the problem.

b) Test $H_0 : \sigma = 1$ versus $H_1 : \sigma < 1$ at $\alpha = 0.01$ level of significance. Draw a conclusion in the context of the problem.

(10) (3 marks) St-Volk has two pizza/sub shops which deliver in the village. For a randomly selected resident, let X be the number of weeky orders from the first shop and Y be the number of weekly orders from the second shop. Suppose that the joint pmf of X and Y is given by the accompanying table:

		Y		
	$p(x,y)$	0	1	2
	0	0.06	0.03	0.01
X	1	0.20	0.20	0.10
	2	0.10	0.14	0.16

a) Compute the marginal probability distributions of X and Y.

b) Compute the conditional probability mass function of Y given that $X = 2$.

c) Compute the conditional mean of Y given $X = 2$.

d) Compute the correlation between the RV's X and Y.

(11) (3 marks) The good folks of St-Volk practice politeness robustly. Here is one example context : If two drivers approach any intersection in the village at roughly the same time from two perpendicular directions they start urging each other to go first. After some friendly waving (could take minutes) one of the drivers concedes to be the jerk on the occasion and passes first. The 'winning' driver takes the moral victory and feels good for the rest of the day. Amelia is very cultured and unless she is rushing to get to work she usually wins these standoffs: her win rate is 83%.

a) What is the probability that Amelia will win for the 10'th time on the 16'th contested crossing for the month?

b) What is the probability that Amelia will win 10 times on the next 16 contested crossings?

c) What is the expected value and the standard deviation for the number of contested crossing Amelia has to go through before her 50th win?

(12) (3 marks) Rainville is a town with some lousy traffic lights. The times (in seconds) Amelia spends waiting to cross two of the worst traffic lights when going to work have been sampled and generated the following sample averages: for traffic light 1: $\bar{x}_1 = 23$ in $n_1 = 22$ crossings; for traffic light 2: $\bar{x}_2 = 34$ in $n_2 = 25$ crossings. Assume that the crossing times are normally distributed with standard deviations of $\sigma_1 = 7$ and $\sigma_2 = 8$ seconds. Construct a 98% confidence interval for the difference of waiting times. Based on this interval can you claim that the second light takes longer to cross?