DIFFERENTIAL EQUATIONS, H23, TEST 1

(1) (2.5 marks) Find the solution $y(t)$ of the initial value problem

$$y' - \frac{y}{t} = t^2 e^{-t}, \quad y(1) = 0.$$

Compute the limit at $t \to +\infty$ of the solution. Sketch the graph of the solution.

(2) (2.5 marks) A particle of mass m is falling downward through a viscous fluid. There are two forces on the particle, gravity and fluid resistance. The net force is $F = mg - \gamma v^2$. The equation of motion, from Newton's second law is,

$$mv' = mg - \gamma v^2$$

Solve this equation for the velocity of the particle as a function of time if $m = 0.001kg$, $g = 9.8m/s^2$ and the the friction coefficient is $\gamma = 0.98kg/m$ under the initial condition $v(0) = 0$. Compute $\lim_{t\to\infty} v(t)$ and interpret the result in the physical context.

(3) (2.5 marks) Find a solution for the initial value problem

$$(x+y)^2 + (2xy + x^2 - 1)y' = 0, \quad y(1) = 1.$$

(4) (2 marks) Consider the differential equation

$$\frac{dy}{dt} = (\lambda - 1)y - y^3,$$

The set of equilibrium solutions depends on the parameter λ.

a) Determine the value of λ which corresponds to a bifurcation point.

b) For values of λ below and above the bifurcation point plot the directional field of the equation together with several integral curves.

c) Draw the bifurcation diagram of this DE, i.e. plot the location of the critical points versus the parameter λ.

(5) (2.5 marks) Determine the solution of the initial value problem

$$y'' + 2y' + 5y = 0, \quad y(0) = -2,\ y'(0) = 1$$

and plot a sketch of the solution.

(6) (2 marks) Determine the general solution of the differential equation

$$y'' - \frac{3}{t}y' + \frac{4}{t^2}y = 0,$$

given that

$$y_1(t) = t^2$$

is one solution.

(7) (3 marks) Solve the initial value problem

$$y'' + 3y' + 3y = 6e^{-2t} + 12, \quad y(0) = 1,\ y'(0) = 0.$$

Compute the limit at $t \to +\infty$ of the solution. Sketch the graph of the solution.

(8) (3 marks) Solve the initial value problem

$$y'' + 4y = 4\sec(2t), \quad y(0) = 2,\ y'(0) = 1$$

What is the maximal interval on which this solution is valid.