PROBABILITY AND STATISTICS, A23, TEST 1

Name: _____ Student number_____

(1) (2.5 marks) Let A, B and C be three events in a sample space with A and B disjoint and C independent from both A and B. Let $p(A) = 0.5$, $p(B) = 0.2$ and $p(C) = 0.3$.
   i) Determine $p(A \cup B \cup C')$.
   ii) Determine $p(A \cup B \mid C')$.

(2) (2.5 marks) A box contains five coins: two regular fair coins, one fake two-headed coin $p(H) = 1$ and two fake two-tailed coins $p(H) = 0$.

i) You pick a coin at random and toss it. What is the probability that it lands heads up?

ii) You pick a coin at random and toss it, and get heads. What is the probability that it is the two-headed coin?

In your solution use the following notation: C_1 for the event that you choose a regular coin, C_2 for the event that you choose a two-headed coin and C_3 for the event that you choose a two-tailed coin.

(3) (2.5 marks) A car company offers four EV models. The models prices and their percentages of the EV sales for this company are as follows:

Model	A	B	C	D
Price	35,000	40,000	55,000	80,000
Sales %	40%	10%	20%	30%

i) Compute the mean and the standard deviation of the sales price.

ii) Image that the two levels of government give 14,000$ in subsides for puchasing of an EV and that this rebate is applied at the point of sale automatically. What would be the expected value and the standard deviation of the sales price after this rebate.

iii) Compute and draw the graph of the cumulate probability function of the sales price.

(4) (2.5 marks) In her work a full-stack developer uses either Python or Javascript or both. The probability that a project requires Python if this project requires Javascript is 0.4. The probability that a a project requires Javascript if it requires Python is 0.25.

i) Determine the probability that a randomply selected project will require both Python and Javascript.

ii) Determine the probability that a randomly selected project will require Python, but no Javascript.

iii) Determine the probability that a randomly selected project will require Javascript (regardless if Python is required or not).

(5) (2.5 marks) A college math teacher estimates that 75% of homework problems are solved with the help of AI. The AI used by the students solves the problems correctly 60% of the time. The students solve the problems correctly without AI 50% of the time.

i) What is the probability that a randomly selected math homework problem will be solved correctly?

ii) The teacher has a random sample of 67 homework problem solutions. What is the expected value and the standard deviation of the number of incorrectly solved problems for this sample?

iii) What is the probbility that all problems in this sample are solved correctly?

(6) (2.5 marks) i) Prove the Bayes' rule in 'odds form'

$$\frac{p(H|E)}{p(H'|E)} = \frac{p(H)}{p(H')} \times \frac{p(E|H)}{p(E|H')}$$

ii) In the formula above E stands for *Evidence* and H stands for *Hypothesis*. Write the formula in the case the *Evidence* and the *Hypothesis* are independent. Next write a sentence (or two) explaining why the Bayes rule becomes trivial in this case.

(7) (2.5 marks) Which of the following events has the higher probability:
A : At least one 6 appears when 6 fair dice are rolled.
B : At least two 6's appear when 12 fair dice are rolled.

(8) (2.5 marks) There are 950 households in a town. Specifically, there are 100 households with one member, 200 households with 2 members, 300 households with 3 members, 200 households with 4 members, 100 households with 5 members, and 50 households with 6 members. Thus, the total number of people living in the town is

$$N = 100 \cdot 1 + 200 \cdot 2 + 300 \cdot 3 + 200 \cdot 4 + 100 \cdot 5 + 50 \cdot 6 = 3000$$

i) We pick a household at random, and define the random variable X as the number of people in the chosen household. Find the PMF and the expected value of X.

ii) We pick a person in the town at random, and define the random variable Y as the number of people in the household where the chosen person lives. Find the PMF and the expected value of Y.