PROBABILITY AND STATISTICS, A23, FINAL EXAMINATION

Name: _____   Student number_____

(1) (3 marks) A real estate company estimates that the average house sales price in Mirabel-en-Haut is $430k$\$. Working for a government agency, you sample the sale prices of 20 houses in Mirabel-en-Haut and find a sample average of $461.88k$\$ with sample standard deviation of $42.125k$\$. Does you data contradict the claim of the real estate agency? Use an appropriate hypothesis test assuming the population distribution of house sale prices is normal. Report bounds for the p-value and make sure to draw a conclusion in the context of the problem.

(2) (3 marks) The time it takes a dandelion flower to turn into a seed head is normally distributed. Gaétan is a horticulturistst from Petit St-Charles and he claims that the standard deviation of this time is 2 days for the lawns in the village. To test his claim your observe a random sample of 16 dandelion flowers. The sample standard deviation of the transformation time is 42 hours. Does this sample contradict Gaétan's claim? Formulate and implement a statistical test. Be sure to include bounds for the p-value.

(3) (3 marks) A study of the amount of calcium in drinking water in Saint-Augustin produced the following six readings in *ppm*

$$9.5 \quad 9.6 \quad 9.3 \quad 9.5 \quad 9.7 \quad 9.2$$

Assume that the population is normally distributed.

a) Construct a 98% confidence interval for the population mean.

b) Construct a 98% confidence interval for the population standard deviation.

(4) (3 marks) The lifetime, in years, of the wild turkeys in Saint-Hermas is well modelled by a random variable with probability density function

$$f(x) = \begin{cases} \frac{32}{(x+4)^3} & x > 0 \\ 0 & x \leq 0 \end{cases}$$

a) Find the mean lifetime.
b) Find the variance of lifetimes.
c) Find the cumulative distribution function of the lifetime.
d) Find the ninetieth percentile, P_{90}, of wild turkey lifetimes.

(5) (3 marks) Sainte-Monique has two (country) road intersections. During the summer days cyclists cross intersection NW at a mean rate of 20 per hour. During the summer days the bicycle traffic at intersection SE has a mean rate of 25 per hour. Assume the bicycle crossings follow two independent Poisson processes at the two intersections of Sainte-Monique.

a) Estimate the probability that there are fewer than 112 bicycle crossings at intersection NW in a six-hour time period during the summer.

b) What is the probability that a drone hovering over Sainte-Monique have to wait more than 5 minutes to observe at least two bicycle crossings for both intersection NW and intesection SE?

(6) (3 marks) Rivière-du-Nord passes through the community of La Chapelle. Five specimens of river water upstream from the crossing under A15 produced an average benzene concentration of 3.32 mg/L with standard deviation of 1.17 mg/L. Seven specimens of river water downstream from A15 had an average benzene concentration of 6.83 mg/L with standard deviation of 1.72 mg/L. Assuming that the two populations are normal construct a 99% confidence interval for the difference of benzene concentration before and after the A15 crossing. Does this confidence interval imply that there is a effect of highway A15 on the river flowing under it in terms of benzene contamination at all? Explain briefly.

(7) (3 marks) A police report claims that speeding a serious problem near Mirabel Airport in Sainte-Scholastique. According to the report during weekends 65% of all vehicles are over the posted speed limit. Rejean and Marie-Josée are on highway patrol duty on weekends.

a) Approximate the probability that on a particular weekend out of the next 120 vehicles more than 80 will be speeding.

b) What is the probability that after setting up the radar the 12'th vehicle passing will be the 8'th vehicle speeding?

c) On a typical weekend 450 vehicles have their speed measured by R&M-J. What is the probability that R&M-J will observe more than 15,000 speeding vehicles in a year worth (52 weeks) of weekend patrols?

(8) (3 marks) Christophe lives in Saint-Jérusalem-d'Argenteuil and commutes for work to Saint-Janvier. He conjectures that the average commute takes him 25 minutes. Assume the standard deviation of Christophe's commutes is known to be 150 seconds. Christophe samples 100 commutes and decides to test the hypothesis that the mean commute time is 25 minutes against the alternative that the mean commuting time is not 25 minutes. Assume also that the true average commuting time is 24 minutes and 15 seconds. If the test is made at the 5% level of significance what is its power? Write a sentence in English explaining what this computed power represents in the context of the problem.

(9) (3 marks) In the near pristine parc Bois de Belle Rivière for a sample of 20 days in the winter, the mass ratio of fine to coarse air particles averaged $\bar{x}_1 = 0.51$ with standard deviation of $s_1 = 0.09$, and for a sample of 13 days in the spring the mass ratio average $\bar{x}_2 = 0.62$ with standard deviation of $s_2 = 0.11$. Assume the populations are normally distributed.

a) Implement a test of the hypothesis $H_0 : \sigma_1 = \sigma_2$ versus $H_1 : \sigma_1 < \sigma_2$ at $\alpha = 0.05$ level of significance. Make sure to report a p-value and draw a conclusion in the context of the problem.

b) Implement a test of the hypothesis $H_0 : \mu_1 = \mu_2$ versus $H_1 : \mu_1 < \mu_2$ at $\alpha = 0.05$ level of significance. Make sure to report a p-value and draw a conclusion in the context of the problem.

(10) (3 marks) The time spent by visitors at Parc Du Domaine Vert is well mod-
elled by the random variable X with pdf $p(x) = 2/x^2$, $1 \le x \le 3$, where x
is in hours. Say, the amount of hot chocolate vistors buy at the lodge at the
end of their visit is $Y = X^2$, where Y is in hundreds of mililiters. Check the
'Theorem of the Unconscious Statistian' on this example. Determine $E(X^2)$
in two ways:

  a) Without finding the pdf of $Y = X^2$.
  b) By first computing the pdf of $Y = X^2$.

(11) (3 marks) The village of Saint-Benoît has two Mom-and-Pop eateries which deliver in the village. For a randomly selected resident, let X be the number of weeky orders from the first eatery and Y be the number of weekly orders from the second eatery. Suppose that the joint pmf of X and Y is given by the accompanying table:

		Y	
$p(x,y)$		0	1	2
	0	0.06	0.03	0.01
X	1	0.20	0.20	0.10
	2	0.15	0.16	0.09

a) Compute the marginal probability distributions of X and Y.

b) Compute the conditional probability mass function of Y given that $X = 1$.

c) Compute the conditional mean of Y given $X = 1$. Write a sentence in English interpreting your findings.

d) Compute the correlation between the RV's X and Y.

(12) (3 marks) Saint-Canut is a village with some lousy traffic lights. The times (in seconds) Amelie spends waiting to cross two of the worst traffic lights when going to work have been sampled and generated the following sample averages: for traffic light 1: $\bar{x}_1 = 23$ in $n_1 = 22$ crossings; for traffic light 2: $\bar{x}_2 = 26$ in $n_2 = 25$ crossings. Assuming the crossing times are normally distributed with standard deviations of $\sigma_1 = \sigma_2 = 7$ seconds, test the hypothesis that the second traffic light takes longer to cross at the $\alpha = 0.05$ level of significance. Report the p-value. Draw a conclusion in the context of the problem.