DISCRETE MATHEMATICS, A24, TEST 1

(1) (3 marks) Are the following statement forms logically equivalent: $p \vee q \to r$ and $\sim p \wedge (q \vee r)$? Include a truth table and a few words explaining how the truth table supports your answer.

(2) (3 marks) Is the following logical expression a tautology?

$$(p \wedge \sim q) \vee (p \wedge q) \leftrightarrow p$$

Include a truth table and a few words explaining how the truth table supports your answer.

(3) (2 marks) Write the convrse, inverse, contrapositive and negation of of "If Jesse got an answer of 11.7 for problem 3 than she solved problem 3 correctly".

(4) (3 marks) Consider the argument form

$$p \to \sim q,\ q \to \sim p\ \vdash\ p \vee q$$

Use a truth table to determine whether this form of argument is valid or invalid. Include a few words explaining how the truth table supports your answer.

(5) (3 marks) Consider the set of premises $\{\sim p \to r \wedge \sim s,\ t \to s,\ u \to \sim p,\ \sim w,\ u \vee w$ and the conculsion $\sim t$. Use valid argument forms to deduce the conclusion from the premises, giving a reson for each step.
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(6) (2 marks) Write the negations of the following two statements:

a) $\exists u \in \mathbb{R}$ such that $\forall v \in \mathbb{R}$, $uv < v$.

b) $\forall r \in \mathbb{Q}$, $\exists a, b \in \mathbb{Z}$ such that $a < r \leq b$.

Argue with concrete examples that the original statement or that negation is true.

(7) (3 marks) Are the following arguments valid or invalid? Justify your answer by pointing out which valid form of argument is used or which type of error is present.

a) All real numbers have nonnegative squares. The number i has negative square. Therefore, the number i is not a real number.

b) All prime numbers greater than 2 are odd. The number a is not prime. Therefore, the number a is not odd.

(8) (3 marks) Use predicate logic to prove that the following argument is valid.

$\exists x$ such that$(P(x) \wedge Q(x)), \forall y\,(Q(y) \to R(y) \vdash \exists x$ such that$(P(x) \wedge R(x)$

(9) (2 marks) Prove that the reciprocal of any rational number is irrational.

(10) (2 marks) Use mathematical induction to prove that $3^{2n} - 1$ is divisible by n for every natural number n.

(11) (2 marks) Suppose that $e_0, e_1, e_2, \ldots$ is a sequence defined as follows:

(12) (3 marks) Use the Euclidean algorithm to find $gcd(6664, 765)$ and then express the greatest common divisor you found as a linear combination of the two numbers.