DISCRETE MATHEMATICS, A24, TEST 3

Name: ________________________ Student number______________________

(1) (2 marks) For the two statements bellow write the negation and then present an argument that original statement is true or that the negation is true.
i) $\forall x \in \mathbb{Z}\ \exists y \in \mathbb{Z}$ s.t. $x + y > 0$.
ii) $\exists x \in \mathbb{N}$ s.t. $\forall y \in \mathbb{Z}\ x + y > 0$.

(2) (2.5 marks) Solve the equation $10x + 61 = 49 + 36x$ in $\mathbb{Z}_{59}$.

(3) (2.5 marks) Solve the recurrence relation

$$A(n) = 4(n-1)A(n-1) \text{ for } n \geq 2,\ A(1) = 2$$

(4) (2.5 marks) Prove that for three sets A, B and C the equality

$$(A \cap B) \cup C = A \cap (B \cup C)$$

holds iff $C \subseteq A$.

(5) Prove that $\mathcal{P}(A) \cup \mathcal{P}(B) \subseteq \mathcal{P}(A \cup B)$ where A and B are arbitrary sets.

(6) (2.5 marks) Let D be the relation defined on $\mathbb{Z}$ as follows: $\forall m, n \in \mathbb{Z}$

$$\{m \; D \; n\} \; \leftrightarrow \; 7|(m^3 - n^3)$$

Prove that this relation is an equivalence relation and describe the distinct equivalence classes of this relation.

(7) (2.5 marks) Let ρ be a binary relation on a set S. Than a binary relation called the inverse of ρ, denoted by ρ^{-1} is defined by $x\rho^{-1}y \leftrightarrow y\rho x$.

Now let (S, ρ) be a poset. Show that $(S, \rho^{-1}))$ is also a poset.

(8) (3 marks) Let $f : S \to T$ and $g : T \to U$ be functions. Prove or disprove the following assertions:
a) If $g \circ f$ is an injection, so is g.
b) If $g \circ f$ is a surjection, so is f.
c) If $g \circ f$ is a bijection, so are both f and g.

(9) (2.5 marks) Determine the coefficient of x^3y^3 in the expansion of $(2x - y - 3)^8$.

(10) (2.5 marks) i) Under what conditions on n and m is the bipartite graph $K_{m,n}$ planar?

ii) Confirm Euler's formula for $K_{2,4}$. You might want to draw a picture to explain your conclusions.

(11) (2.5 marks) Let G be a simple graph. Prove that G is a tree iff G is connected and the addition of one arc to G results in a graph with exactly one cycle.

(12) (3 marks) Consider the binary relation $\rho = \{(1,2),(1,4),(2,1),(2,2),(2,3),(3,3),(4,3)\}$ on the set $S = \{1,2,3,4\}$.

a) Draw the associated directed graph and the adjacency matrix.

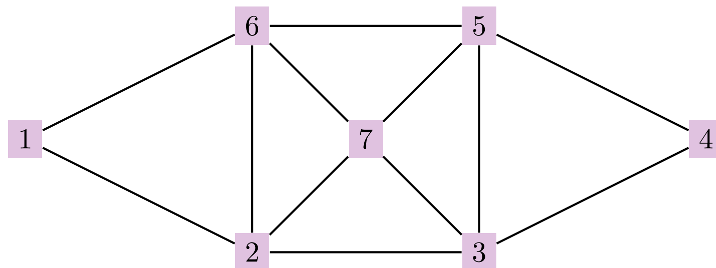
b) Determine the transitive closure of ρ by computing the reachability matrix (show details).

(13) (2.5 marks) Recall that K_n denotes the simple, complete graph with n vertices.
i) For what values of n does an Euler path exist in K_n?
ii) For what values of n does a Hamiltonian circuit exist in K_n?

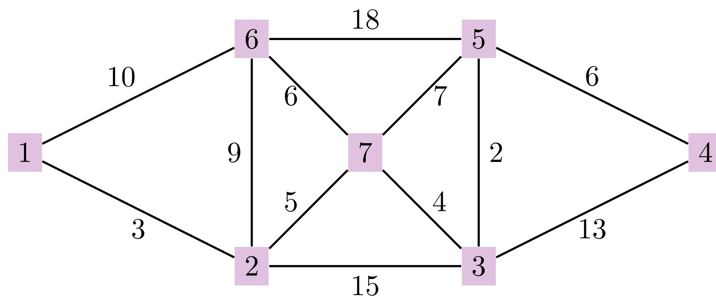
(14) (2.5 marks) For the graph below:

i) Write the nodes in a depth-first search starting at node 1 and following an alphabetical order.

ii) Write the nodes in a breadth-first search starting at node 1 and following an alphabetical order.



(15) (3 marks) For the weighted graph below, while describing every step in the algorithm you are using, find the shortest distance between node 1 and node 4.



(16) (2 marks) For the graph from problem 15 describe step-by-step construction of a minimal spanning tree.