PROBABILITY AND STATISTICS, A24, TEST 1

Name: _____ Student number_____

(1) (2 marks) A bag contains 4 red, 6 blue and 2 green marbles. Three marbles are randomly selected from the bag without replacement.

i) What is the probability that the three marbles are of the same color?

ii) What is the probability that the three marbles selected are all red if they are of the same color?

(2) (3 marks) Let A, B and C be three events in a sample space with
a) A disjoint from both B and from C and,
b) B and C are independent from each other and,
c) $p(A) = 0.45$, $p(B) = 0.2$ and $p(C) = 0.3$.

Based on this information:

i) Determine $p(A \cap B' \mid C')$.
ii) Determine $p(A \cup B \mid C')$.
iii) Are all three events A, B and C independent as a group? Explain!

(3) (3 marks) Yvan commutes to work on the Lautentian autoroute. Let X be the random variable modelling the number of speed traps he encounters on any given day. Data collected over more than a decade shows X has the following probability mass function:

X	0	1	2	3	4
$p(x)$	0.44	0.28	0.19	0.07	0.02

i) Compute the expected value and the standard deviation of the number of speed traps encountered.

ii) Yvan is a law obiding citizen but on occasion he might drift into enjoing the beautiful scenery. This results in a 0.005 chance of being stopped at any of these speed traps while driving over the speed limit. Let Z the number of speeding tickets Yvan gets in a year which has 252 commutes to work. What is the expected value of Z and what is the standard deviation.

(4) (2.5 marks) Phyllis feels numbness and tingling in some of the digits on her left foot. She visits her newly minted family physician, Dr. Newbee, and presents the symptoms. Dr. Newbee googles the symptoms and finds that 2 mutually exclusive conditions, A and B, are possible. The prevalence of these conditions in the general population is as follows: $p(A) = 0.45$ and $p(B) = 0.09$. 36% of patients with condition A show the symptoms, 90% of patients with condition B show the symptoms and 12% of patients who are not afflicted with either of the two conditions have these symptoms.
a) What is the probability that Phyllis suffers from condition A or condition B?
b) What is the probability that Phyllis is not afflicted with either of the two conditions despite the symptoms?

(5) (2.5 marks) In a sample of 1425 homework problems Sandy, college math teacher, has found 251 solutions identical to solutions outputted by AI. The Dean who wants data to evaluate the use of AI asks for a random sample of 20 homework problems from this set.

i) What is the expected value and the standard deviation of homework problems in the sample of 20 which have been solved by AI.

ii) Estimate the probability that two of more randomly selected math homework problems in this sample has been solved by AI?

iii) The Dean start checking the homework problems in the sample one-by-one against AI solutions. Estimate the probability that the Dean will find the first AI solution on the ninth problem he checks.

(6) (2.5 marks) Each day during the summer vacation period Ioannis either goes on a bike ride or goes on a jogging run or both. The probability that Ioannis goes on a bike ride given that he goes jogging is 0.15. The probability that Ioannnis goes jogging given that he has done a bike ride is 0.24. Determine the probablity that on a randomly selected summer day Ioannis goes both jogging and biking.

(7) (2.5 marks) Consider a well-shuffled standard deck of 52 cards.

i) What is the probability that the top two cards are spades?

ii) What is the probability that the top two cards are hearts and the bottom two cards are hearts as well?

iii) What is the probability that all jacks are next to each other, all queens are next to each other and all kings are next to each other?

(8) (2 marks) i) For any three events A, B and C of nonzero probability prove that.

$$p(A \cup B|C) = p(A|C) + p(B|C) - p(A \cap B)|C)$$

ii) How does this equality simplify if the three events are independent?