

Linear Algebra — Class Exercise 1

Systems of Linear Equations

1. Write the augmented matrix of

$$3z - 2y = 5 + x$$

$$4x = 1 - z$$

$$y + 2x = 0$$

using the variable order x, y, z .

2. Starting from

$$\left[\begin{array}{ccc|c} 1 & 3 & -2 & 4 \\ 3 & 8 & 1 & 14 \\ 0 & 1 & 5 & -2 \end{array} \right]$$

apply, in this order: $R_2 \rightarrow R_2 - 3R_1$, then $R_1 \leftrightarrow R_3$, then $R_2 \rightarrow R_2 + R_1$. Write the matrix after each step.

3. Find the false step in this argument, and say why it fails.

Claim. A system of two linear equations in three unknowns always has infinitely many solutions.

1. Each equation $ax + by + cz = d$ describes a plane in space.
 2. Two distinct planes meet in a line.
 3. A line contains infinitely many points.
 4. So every such system has infinitely many solutions.
4. Give a system of three linear equations in three unknowns which is inconsistent, in which no equation is a multiple of any other, and in which *any two* of the three equations, taken on their own, form a consistent system. Describe its solution set.