

Linear Algebra — Class Exercise 10

Inverses at Work and the Inversion Algorithm

1. One Inverse, Several Systems. Let

$$A = \begin{bmatrix} 3 & 4 \\ 2 & 3 \end{bmatrix}, \quad \mathbf{b}_1 = \begin{bmatrix} 11 \\ 8 \end{bmatrix}, \quad \mathbf{b}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad \mathbf{b}_3 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}.$$

- (a) Find A^{-1} and verify it.
- (b) Solve the three systems $A\mathbf{x} = \mathbf{b}_1$, $A\mathbf{x} = \mathbf{b}_2$ and $A\mathbf{x} = \mathbf{b}_3$ with a *single* matrix product, by placing $\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3$ side by side as the columns of one matrix.
- (c) Check each of the three solutions in the original system.
- (d) Without solving anything, explain why $A\mathbf{x} = \mathbf{b}$ has exactly one solution for *every* $\mathbf{b} \in \mathbb{R}^2$.

2. Solving for a Matrix Between A and A^{-1} . Let

$$A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix}.$$

- (a) Solve the equation $AXA^{-1} = B$ for X in terms of A and B . Say at every step whether you left-multiply or right-multiply.
- (b) Find A^{-1} , and then compute X for the given matrices.
- (c) Check your answer by computing AXA^{-1} .
- (d) A student claims that “ A and A^{-1} cancel, so $X = B$ ”. Show that this is false for the given matrices, and explain where the reasoning goes wrong.

3. Inverting a 3×3 Matrix. Let

$$C = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 4 \\ 2 & 1 & -1 \end{bmatrix}.$$

- (a) Find C^{-1} by row-reducing the block matrix $[C \mid I]$. Name every row operation you use, and separate the forward phase (zeros below the pivots) from the backward phase (zeros above the pivots).
- (b) At which point of the reduction can you be sure that C is invertible? Why?
- (c) Verify your answer by computing CC^{-1} .