Linear Algebra — Class Exercise 13

Properties of Determinants

1. **Determinants Without Matrices.** A and B are 4×4 matrices with $\det(A) = 2$ and $\det(B) = -3$. Find

   (a) $\det\big(A^{-1}(3B)^T\big)$,

   (b) $\det(A^3B^2)$,

   (c) $\det(-B)$.

   Name the rule you use at each step. Do you need to know any entry of A or B?

2. **The Scalar Rule.** Three students make the following claims. In each case find the mistake and give the correct value.

   (a) A is 4×4 with $\det(A) = 3$. "Then $\det(-A) = -3$."

   (b) A is 3×3 with $\det(A) = 5$. "Then $\det\big((2A)^{-1}\big) = 2^3/5 = 8/5$."

   (c) A is 4×4 with $\det(A) = -2$. "Then $\det\big((-2A)^{-1}\big) = 1/(-2 \cdot (-2)) = 1/4$."

3. **What $\det = 0$ Does Not Tell You.** Let $N = \begin{bmatrix} 1 & 2 & 1 \\ 2 & k & 2 \\ 1 & 2 & k \end{bmatrix}$.

   (a) Find the values of k for which N is singular.

   (b) A student says: "At $k = 1$, $\det(N) = 0$, so $N\mathbf{x} = \mathbf{b}$ has no solution." Show that this is false by solving $N\mathbf{x} = \mathbf{b}$ for $\mathbf{b} = (1, 2, 1)^T$, and show that for $\mathbf{b} = (1, 2, 2)^T$ there really is no solution.

   (c) Repeat at $k = 4$ with $\mathbf{b} = (1, 2, 3)^T$ and $\mathbf{b} = (1, 0, 3)^T$.

   (d) What does $\det(N) = 0$ tell you about $N\mathbf{x} = \mathbf{b}$, and what must you do to find out more?