Linear Algebra — Class Exercise 14

The Adjugate and Cramer's Rule

1. **Inverse by the Adjugate.** Let $A = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 3 & -1 \\ 0 & 2 & 1 \end{bmatrix}$.

   (a) Compute $\det(A)$ from the cofactors of row 1.

   (b) Compute the other six cofactors, and write down $\operatorname{cof}(A)$ and $\operatorname{adj}(A)$.

   (c) Write A^{-1}, and check one row of $A \operatorname{adj}(A)$.

2. **A 3×3 System by Cramer's Rule.** Solve

$$\begin{aligned} x + y + 2z &= 9 \\ 2x - y + z &= 3 \\ x + 3y - z &= 4 \end{aligned}$$

   by Cramer's rule: compute $\det(A)$, $\det(A_1)$, $\det(A_2)$ and $\det(A_3)$, and then check your solution in all three equations.

3. **A Formula, Not a Number.** Consider the system

$$\begin{aligned} x + ky &= 2 \\ kx + 4y &= 1 \end{aligned}$$

   (a) For which values of k does the system have a unique solution?

   (b) For those k, use Cramer's rule to find x and y as expressions in k.

   (c) For each value of k found in (a) where the solution is not unique, use elimination to decide whether the system has no solution or infinitely many.