Linear Algebra — Class Exercise 16

The Dot Product and Projections

1. For which k are $(k, 3, -2)$ and $(k, k, 5)$ orthogonal?

2. Given $\|\mathbf{u}\| = 5$, $\|\mathbf{v}\| = 4$ and $\mathbf{u} \cdot \mathbf{v} = 10$, find $\|\mathbf{u} + \mathbf{v}\|$, $\|\mathbf{u} - \mathbf{v}\|$, the angle between $\mathbf{u}$ and $\mathbf{v}$, and $(2\mathbf{u} - \mathbf{v}) \cdot (\mathbf{u} + 3\mathbf{v})$.

3. Find all three angles of the triangle $A(1, 3, -1)$, $B(-1, 4, 1)$, $C(0, 4, -1)$. What should they add to?

4. Decompose $\mathbf{u} = (1, 4, -2)$ relative to $\mathbf{v} = (2, -1, 2)$, and carry out both checks.