Linear Algebra — Class Exercise 17

Projections, Cross Product and Triple Product

1. Find the distance from $P(-5, 2, -4)$ to the line through the origin with direction $\mathbf{v} = (2, -2, 1)$, and find the point Q on the line that is closest to P. Check that $\overrightarrow{QP}$ is perpendicular to $\mathbf{v}$.

2. Find the area of the parallelogram with sides $(5, 2)$ and $(-3, 4)$ in the plane, once by the cross product and once by a 2×2 determinant.

3. Find the volume of the parallelepiped with edges $(2, 0, 1)$, $(1, 3, -1)$, $(0, 2, 4)$. What changes if the first two edges are swapped?

4. For which k are $(1, k, 2)$, $(3, -1, 1)$, $(2, 4, 0)$ coplanar? For that k, write the first vector as a combination of the other two.