Linear Algebra — Class Exercise 18

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1. In a town, 90% of the people who vote in one municipal election also vote in the next one, and 10% do not. Of the people who do not vote in one election, 40% vote in the next one and 60% do not. Take state 1 to be *votes* and state 2 to be *does not vote*; one step is one election.

   (a) Write the transition matrix P and check that it is stochastic.

   (b) Compute P^2.

   (c) Interpret each of the four entries of P^2 in the context of the town.

   (d) In the most recent election, 60% of the town voted, so $\mathbf{x}_0 = (0.6,\ 0.4)^T$. Compute $P^2\mathbf{x}_0$ and interpret its entries.

2. Voters identify with party A, party B, or as independent (I). Each election, of A's supporters 60% stay, 10% move to B and 30% become independent; of B's, 10% move to A, 70% stay, 20% become independent; of independents, 10% join A, 10% join B and 80% stay. Today the split is $(0.4, 0.4, 0.2)$. Find the split after one election, and the long-run split.