Linear Algebra — Class Exercise 3

Gauss–Jordan and the Reduced Row Echelon Form

1. For each matrix, say whether it is in *reduced row echelon form*, in *row echelon form only*, or in *neither*. When a condition fails, name it and say which entry is at fault.

$$M_1 = \begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & 2 \end{bmatrix} \qquad M_2 = \begin{bmatrix} 0 & 1 & -4 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad M_3 = \begin{bmatrix} 1 & 3 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 4 \end{bmatrix}$$

$$M_4 = \begin{bmatrix} 1 & -2 & 4 \\ 0 & 3 & 1 \\ 0 & 0 & 1 \end{bmatrix} \qquad M_5 = \begin{bmatrix} 1 & 0 & -1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 5 \end{bmatrix} \qquad M_6 = \begin{bmatrix} 1 & 0 & 2 & 4 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2. Solve

$$\begin{aligned} 3x + y + 2z &= 11 \\ 2x + 3y - z &= -2 \\ -2x + y + 4z &= 7 \end{aligned}$$

by Gauss–Jordan, carrying the reduction all the way to reduced row echelon form. Name every row operation, and check your answer in the original system.

Hint. The $(1,1)$ entry is a 3. Before scaling row 1 by $\frac{1}{3}$ and dragging thirds through the whole problem, look at what row 2 has in that column.

3. Solve

$$\begin{aligned} x + y + z &= 2 \\ 2x - y + 3z &= -1 \\ x + 2y - z &= 7 \\ 3x \qquad + 4z &= 1 \end{aligned}$$

by Gauss–Jordan. Four equations, three unknowns — how many solutions does it have, and what does the row of zeros tell you? Check your answer in the original system.