

Linear Algebra — Class Exercise 6

Matrix Algebra and Matrix Types

1. Let A be a 3×2 matrix, B a 2×3 matrix, and C a 3×2 matrix. For each of the following expressions, state whether it is defined. If it is defined, give the size of the resulting matrix; if it is undefined, explain why.

(a) $2A - 5C$

(b) $A + B$

(c) $A + B^T$

(d) $(3B - 4A^T)^T + 2C$

Hint. Addition requires identical dimensions; transposition flips $m \times n$ to $n \times m$.

2. Solve the following equation for the unknown 2×2 matrix X :

$$2(X - A^T) + B = X - 3I_2$$

where

$$A = \begin{bmatrix} 3 & -2 \\ 1 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} -1 & 5 \\ 2 & 0 \end{bmatrix}, \quad I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Hint. Isolate X algebraically before substituting any matrices.

3. Decompose the matrix

$$M = \begin{bmatrix} 5 & -3 & 2 \\ 7 & 1 & -4 \\ -6 & 2 & 0 \end{bmatrix}$$

into the sum of a symmetric matrix S and a skew-symmetric matrix K , so that $M = S + K$. Write down S and K explicitly, and verify that $S^T = S$, $K^T = -K$, and $S + K = M$.

Hint. Use $S = \frac{1}{2}(M + M^T)$ and $K = \frac{1}{2}(M - M^T)$.

4. Consider the upper triangular matrix parameterized by $k \in \mathbb{R}$:

$$A = \begin{bmatrix} 2k - 1 & k - 2 & 0 \\ 0 & k + 1 & k^2 - 4 \\ 0 & 0 & 3k - 3 \end{bmatrix}$$

- (a) For which value(s) of k is A a **diagonal** matrix?
- (b) For which value(s) of k is A a **symmetric** matrix? Explain why this condition is identical to part (a).
- (c) Can A ever be skew-symmetric? Justify your answer in one or two sentences.