Class Exercise 2 - Solutions

1. **Swiss Cheese**

As food hygiene standards have improved, fewer holes are appearing naturally in Swiss cheeses. Dairies are now forced to artificially create them. The table shows the number of holes per slice and the corresponding number of slices observed:

Number of Holes	Number of Slices		
x	f	fx	fx^2
0	4	0	0
4	51	204	816
6	18	108	648
8	52	416	3328
10	23	230	2300
12	8	96	1152
14	4	56	784
	160	1110	9028

a. Calculate the average number of holes per slice.

Solution

$$\overline{x} = \frac{\sum fx}{n} = \frac{1110}{160} = 6.9375 \text{ holes}$$

b. Calculate the variance and standard deviation for the number of holes per slice. Include units in your answers.

Solution

$$\begin{aligned} s^2 &= \frac{1}{n-1}\left[\sum fx^2 - \frac{(\sum fx)^2}{n}\right] \\ &= \frac{1}{160-1}\left[9028 - \frac{(1110)^2}{160}\right] = \frac{1}{159}[9028 - 7700.6258] = 8.3483 \text{ holes}^2 \end{aligned}$$

$$s = \sqrt{s^2} = \sqrt{8.3483} = 2.889 \text{ holes}$$

c. Which is the more appropriate graphical representation for this data: a bar chart or a histogram? Explain briefly **and** describe the shape of the distribution of holes per slice.

Solution

Since the number of holes is a discrete random variable, the most appropriate graphical representation for the data is a **bar chart**. The distribution bimodal.

2. **Flamingos**

Flamingoes that migrate to spend their winters in the sun age more slowly than the ones that stay put. A group of flamingoes was observed, and the number of days they spent migrating during the year was recorded.

Days Spent Migrating	Number of Flamingoes			
	f	m	fm	fm^2
$[0, 20)$	12	10	120	1200
$[20, 40)$	28	30	840	25200
$[40, 60)$	45	50	2250	112500
$[60, 80)$	35	70	2450	171500
$[80, 100)$	20	90	1800	162000
	140		7460	472400

a. Calculate the mean number of days spent migrating for this group of flamingoes.

Solution

$$\overline{x} = \frac{\sum fm}{n} = \frac{7460}{140} = 53.2857 \text{ days}$$

b. Calculate the variance and standard deviation for the number of days spent migrating. Include units in your answers.

Solution

$$s^2 = \frac{1}{n-1}\left[\sum fm^2 - \frac{(\sum fm)^2}{n}\right]$$
$$= \frac{1}{140-1}\left[472400 - \frac{(7460)^2}{140}\right] = \frac{1}{139}[74888.5714] = 538.7667 \text{ days}^2$$

$$s = \sqrt{s^2} = \sqrt{538.7667} = 23.2113 \text{ holes}$$

c. The empirical rule states that at least 95% of the data can be found within two standard deviations of the mean (i.e. $\overline{x} \pm 2s$). Determine and interpret interval for the number of days that we would expect 95% of flamingos to spend migrating during the year.

Solution

$$\overline{x} \pm 2 \cdot s$$
$$53.2857 \pm 2 \cdot (23.2113)$$
$$(6.863,\ 99.7083)$$

Interpretation: Assuming the distribution of migration days is roughly bell-shaped, about 95% of individual flamingoes in this group would be expected to spend between 7 and 100 days migrating in a year.