Class Exercise 6 Solutions

1. **Dr. Newbee**

Phyllis feels numbness and tingling in some of the digits on her left foot. She visits her newly minted family physician, Dr. Newbee, and presents the symptoms. Dr. Newbee googles the symptoms and finds that two not mutually exclusive conditions, A and B, are possible.

The prevalence of these conditions in the general population is as follows: $P(A) = 0.45$, $P(B) = 0.39$ and $P(A \cap B) = 0.04$. Moreover, 36% of patients with condition A only show the symptoms, 90% of patients with condition B only show the symptoms, 99% of the patients afflicted by both conditions show the symptoms and 12% of patients who are not afflicted with either of the two conditions have these symptoms.

(a) What is the probability a randomly selected person is only afflicted with condition A?

Solution

Let OA = the person is only afflicted with condition A

$$P(OA) = P(A) - P(A \cap B) = 0.45 - 0.04 = 0.41$$

(b) What is the probability a randomly selected person is only afflicted with condition B?

Solution

Let OB = the person is only afflicted with condition B

$$P(OB) = P(B) - P(A \cap B) = 0.39 - 0.04 = 0.35$$

(c) What is the probability a randomly selected person is afflicted with neither condition A nor B?

Solution

Want $P(A' \cap B') = P(A \cup B)'$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.45 + 0.39 - 0.04 = 0.80$$
$$\therefore \quad P(A \cup B)' = 1 - P(A \cup B) = 1 - 0.80 = 0.20$$

(d) What is the probability a randomly selected person is shows symptoms?

Solution

Let $S =$ the person shows symptoms

Want $P(S)$, and we know that $P(S \mid OA) = 0.36, P(S \mid OB) = 0.90, P(S \mid A \cap B) = 0.99,$ and $P(S \mid (A' \cap B')) = 0.12$

Using the rule of total probability we have

$$\begin{aligned} P(S) &= P(S \mid OA) \cdot P(OA) + P(S \mid OB) \cdot P(OB) + P(S \mid A \cap B) \cdot P(A \cap B) \\ &\quad + P(S \mid A' \cap B') \cdot P(A' \cap B') \\ &= P(S \mid OA) \cdot P(OA) + P(S \mid OB) \cdot P(OB) + P(S \mid A \cap B) \cdot P(A \cap B) \\ &\quad + P(S \mid (A \cup B)') \cdot P(A \cup B)' \\ &= 0.36 \cdot (0.41) + 0.90 \cdot (0.35) + 0.99 \cdot (0.04) + 0.12 \cdot (0.20) \\ &= 0.5262 \end{aligned}$$

(e) Suppose that Physllis shows symptoms. What is the probability that she suffers from only condition A?

Solution

$$P(OA \mid S) = \frac{P(OA \cap S)}{P(S)} = \frac{P(S \mid OA) \cdot P(OA)}{P(S)} = \frac{0.36 \cdot (0.41)}{0.5262} = 0.2805$$

(f) What is the probability that Phyllis is not afflicted with either of the two conditions A, B despite the showing symptoms?

Solution

$$P(A' \cap B' \mid S) = \frac{P((A' \cap B') \cap S)}{P(S)} = \frac{P(S \mid A' \cap B') \cdot P(A' \cap B')}{P(S)} = \frac{0.12 \cdot (0.20)}{0.5262} = 0.0456$$

(g) Determine the probability that a randomly chosen person from the population is not afflicted by both conditions, given that he or she is afflicted by at least one of the two conditions.

Solution

Want $P(A' \cap B' \mid A \cup B)$. This is equal to $1 - P(A \cap B \mid A \cup B)$

$$P(A \cap B \mid A \cup B) = \frac{P((A \cap B) \cap (A \cup B))}{P(A \cup B)} = \frac{P(A \cap B)}{P(A \cup B)} = \frac{0.04}{0.80} = 0.05$$

$$\therefore \quad P(A' \cap B' \mid A \cup B) = 1 - P(A \cap B \mid A \cup B) = 1 - 0.05 = 0.95$$