Assignment 1 - Solutions

1. **Shaken. Not Stirred**

A new study published in the *Medical Journal of Australia* called *"License to Swill"* has found that James Bond has a "severe alcohol use disorder". In *Quantum of Solace*, he downs six vesper martinis on an aeroplane — enough to kill some people. The number of alcoholic beverages consumed by Bond in 45 missions is shown below:

2	5	6	6	7	7	7	8	8	8	9	9	9	9	9
10	10	10	11	11	11	12	12	12	13	13	13	14	14	14
15	15	15	16	16	16	17	17	17	18	18	18	19	19	24

(a) Organize the data into **seven classes** and make a table showing: class limits, class boundaries, the frequencies, relative frequencies, the less-than cumulative frequencies (LTCF), the LTCF's in decimal, the more-than cumulative frequencies (MTCF), and the MCTF's in decimal.

Solution

$$Class\,width = \frac{Highest - Lowest}{\#\,of\,classes} = \frac{24-2}{7} = 3.14 \quad \Rightarrow \quad 4$$

Class Limits	Class Boundaries	Freq.	Rel. Freq. (dec.)	LTCF	LCTF (dec.)	MTCF	MTCF (dec.)
2 – 5	1.5 – 5.5	2	0.0444	2	0.0444	45	1.0000
6 – 9	5.5 – 9.5	13	0.2889	15	0.3333	43	0.9556
10 – 13	9.5 – 13.5	12	0.2667	27	0.6000	30	0.6667
14 – 17	13.5 – 17.5	12	0.2667	39	0.8667	18	0.4000
18 – 21	17.5 – 21.5	5	0.1111	44	0.9778	6	0.1333
22 – 25	21.5 – 25.5	1	0.0222	45	1.0000	1	0.0222
26 – 29	25.5 – 29.5	0	0.0000	45	1.0000	0	0.0000
		45					

(b) What is the probability that Bond consumes at least 14 drinks in on a mission?

Solution

Let X = the number of drinks that Bond consumes on a mission. From the MTCF column of the table:

$$P(X \geq 14) = 0.4$$

(c) What is the probability that Bond consumes at most 9 drinks or at least 22 drinks on assignment?

Solution

$$P(X \leq 9 \text{ or } X \geq 22) = P(X \leq 9) + P(X \geq 22) = 0.3333 + 0.0222 = 0.3555$$

(d) What is the probability that Bond consumes between 10 and 21 drinks (inclusive) on the job?

Solution

$$P(10 \leq X \leq 21) = \frac{12 + 12 + 5}{45} = \frac{29}{45} = 0.6444$$

2. **Mocker Swallowtail Butterflies**

To avoid predation, female mocker swallowtail butterflies can be born in one of at least 14 different guises that all resemble poisonous butterflies. The males all look the same. The wingspans of 42 swallowtail butterflies in millimetres are shown below.

50	58	59	60	65	66	67	67	68	74	74	75	75	77
77	77	78	78	81	83	85	87	88	88	88	88	89	89
89	89	89	89	89	90	90	90	90	91	91	91	92	92

(a) Organize the data into **six classes** and make a table showing: class limits, class boundaries, the frequencies, relative frequencies (in %), the less-than cumulative frequencies (LTCF), the LTCF's in decimal, the more-than cumulative frequencies (MTCF), and the MCTF's in decimal.

Solution

$$Class\, width = \frac{Highest - Lowest}{\#\, of\, classes} = \frac{92 - 50}{6} = 7 \quad \Rightarrow \quad 8$$

Class Limits	Class Boundaries	Freq.	Rel. Freq. (%)	LTCF	LCTF (dec.)	MTCF	MTCF (dec.)
50 – 57	49.5 – 57.5	1	2.38	1	0.0238	42	1.0000
58 – 65	57.5 – 65.5	4	9.52	5	0.1190	41	0.9762
66 – 73	65.5 – 73.5	4	9.52	9	0.2143	37	0.8810
74 – 81	73.5 – 81.5	10	23.81	19	0.4524	33	0.7857
82 – 89	81.5 – 89.5	14	33.33	33	0.7857	23	0.5476
90 – 97	89.5 – 97.5	9	21.43	42	1.0000	9	0.2143
		42					

(b) How many butterflies in the sample have a wingspan of 74 mm or more?

Solution

$$33 \text{ butterflies}$$

(c) What is the probability that a randomly selected butterfly has a wingspan at most 65 mm or least 90 mm?

Solution

Let X = the wingspan length of a randomly selected butterfly

$$P(X \leq 65 \text{ or } X \geq 90) = P(X \leq 65) + P(X \geq 90) = 0.1190 + 0.2143 = 0.3333$$

(d) Based on the frequency distribution, which wingspan interval is the least likely for a randomly selected butterfly? Explain your reasoning.

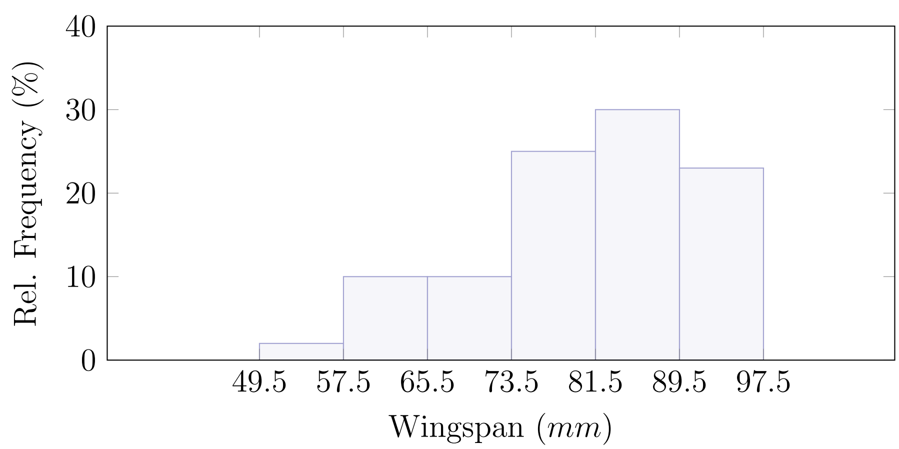
Solution

The least likely interval is for a randomly selected butterfly to occur in, is $50 - 57$ mm. Justification: it has the smallest relative frequency.

(e) Decide between a bar graph and a histogram. Then sketch a graph of the relative frequencies (in %) for the wingspan data. Label your axes, and comment on the shape of the distribution.

Solution

Since wingspan is a continuous random variable, the appropriate graphical representation of the data is a **histogram**.



Comment: The distribution is unimodal; skewed left.

3. **Kea Birds**

In New Zealand, kea birds are so fond of stealing traffic cones that authorities have been forced to set up roadside "gyms" to keep the birds occupied. The number of cones stolen per week by a mischievous flock of kea was recorded over 48 weeks, as shown below:

3	4	5	6	6	7	7	8	8	9	9	9	10	10	10	11
11	11	12	12	12	12	13	13	13	13	13	14	14	14	15	15
15	15	15	15	15	16	16	16	17	17	17	17	17	17	20	20

(a) Organize the data into **six classes** and make a table showing: class limits, class boundaries, the frequencies, relative frequencies, the less-than cumulative frequencies (LTCF), and the more-than cumulative frequencies (MTCF).

Solution

$$Class\,width = \frac{Highest - Lowest}{\#\,of\,classes} = \frac{20-3}{6} = 2.83 \quad \Rightarrow \quad 3$$

Class Limits	Class Boundaries	Freq.	Rel. Freq.	LTCF	MTCF
$3-5$	$2.5-5.5$	3	0.0625	3	48
$6-8$	$5.5-8.5$	6	0.1250	9	45
$9-11$	$8.5-11.5$	9	0.1875	18	39
$12-14$	$11.5-14.5$	12	0.25	30	30
$15-17$	$14.5-17.5$	16	0.3333	46	18
$18-20$	$17.5-20.5$	2	0.0417	48	2
		48			

(b) What is the probability that the kea flock steals 9 or more cones in a given week?

Solution

Let X = the number of traffic cones that a kea flock steals in a week.

$$P(X \geq 9) = \frac{39}{48} = 0.8125$$

(c) What is the probability that the kea flock steals at most 14 in a given week?

Solution

$$P(X \leq 14) = \frac{30}{48} = 0.625$$

(d) Which is event is more likely: that a kea flock steals exactly 12 cones in a week, or that a flock of keas steals exactly 17 cones in a week? Justify your answer with some calculations.

Solution

$$P(X=12)=\frac{4}{48} \qquad\qquad P(X=17)=\frac{6}{48}$$

$\because P(X=17)>P(X=12)$ the more likely scenario is 17 cones/week

4. **Balloons**

Artist Masayoshi Matsumoto has taken balloon modelling to a whole new level. Watch how he makes them **here**

The number of balloons used to make 45 sculptures are shown below

11	12	14	16	18	20	22	23	26	28	30	32	33	33	34
34	35	35	35	35	35	35	36	36	38	39	39	44	46	48
49	51	53	53	54	54	62	64	66	70	72	74	92	94	95

(a) Organize the data into **seven classes** and make a table showing: class limits, class boundaries, the frequencies, relative frequencies (in decimal), the less-than cumulative frequencies (LTCF), the LTCF's in decimal, the more-than cumulative frequencies (MTCF), and the MCTF's in decimal.

Solution

$$Class\,width=\frac{Highest-Lowest}{\#\,of\,classes}=\frac{95-11}{7}=12 \quad\Rightarrow\quad 13$$

Class Limits	Class Boundaries	Freq.	Rel. Freq. (dec.)	LTCF	LCTF (dec.)	MTCF	MTCF (dec.)
11 – 23	10.5 – 23.5	8	0.1778	8	0.1778	45	1.0000
24 – 36	23.5 – 36.5	16	0.3556	24	0.5333	37	0.8222
37 – 49	36.5 – 49.5	7	0.1556	31	0.6889	21	0.4667
50 – 62	49.5 – 62.5	6	0.1333	37	0.8222	14	0.3111
63 – 75	62.5 – 75.5	5	0.1111	42	0.9333	8	0.1778
76 – 88	75.5 – 88.5	0	0.0000	42	0.9333	3	0.0667
89 – 101	88.5 – 101.5	3	0.0667	45	1.0000	3	0.0667
		45					

(b) What is the probability that a randomly selected sculpture uses at least 24 and at most 62 balloons?

Solution

Let X = the number of balloons used in a sculpture.

$$P(24 \leq X \leq 62) = \frac{29}{45} = 0.6444$$

(c) What is the probability that a randomly selected sculpture does not fall into the the two categories from part (b)?

Solution

$$P(X < 24 \text{ or } X \geq 62) = 1 - \frac{29}{45} = \frac{16}{45} = 0.3556$$

(d) In the long run, if 500 balloon sculptures are made, how many would you expect to use at least 50 balloons?

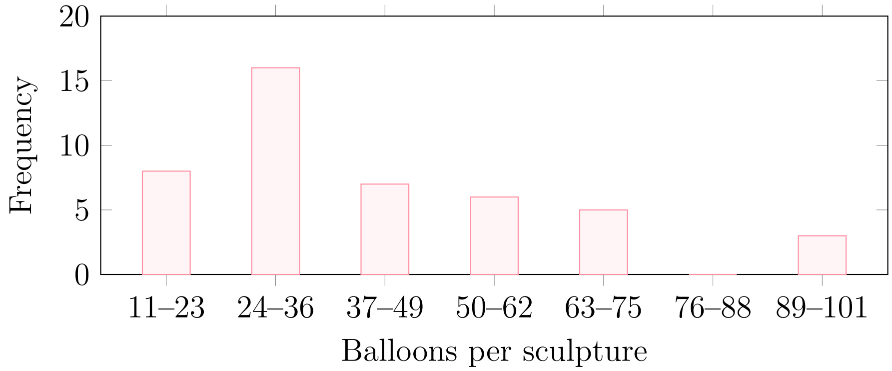
Solution

If we have 500 sculptures, then we would expect $500 \cdot P(X \geq 50) = 500 \times \frac{14}{45} = 155.5556 \approx 156$ sculptures that uses at least 500 balloons.

(e) Decide between a bar graph and a histogram. Then, sketch a graph of the frequencies for the number of balloon used to make a sculpture. Label your axes, and comment on the shape of the distribution.

Solution

Since the number of balloons is a discrete random variable, the appropriate graphical representation is a **bar chart**.



Comment: The distribution is unimodal, skewed right.