

Assignment 3

Instructions

- Deadline: **October 30 (4:00 pm)**
- Format: Hard copy (on paper) only. **Submissions by MIO will not be accepted.**
- Please show all of your work on your submission.
- Notation counts. Poor notation will result in a loss of marks.
- Please leave your answers as exact values. If using decimals, please report your answer to four decimal places.
- You are encouraged to ask your instructor for help, and/or discuss ideas with your classmates. However, you must produce fully explained individual solutions.
- Under no circumstances may you simply copy solutions obtained online or from a classmate.
- In unclear cases, you may be asked to explain your solutions in a Teams meeting, and your work may be refused altogether.

1. Koala

In 2006, thieves planning to steal a koala from a zoo in Australia, had to change their minds after it proved too vicious to be kidnapped. After getting thoroughly mauled, they gave up and stole a crocodile instead.

- (a) A witness reported that a car seen speeding away from the zoo had a number plate that began with a V or W , followed by the digits 4, 7, and 8 in some order, and ending with letters A , C , and E in some order. Determine the number of cars that would need to be examined to ensure that the suspect's vehicle is included.
- (b) In other parts of Australia, a licence plate consists of a sequence of seven symbols: number, letter, letter, letter, number, number, number, where a letter is any one of 26 letters ($A - Z$) and a number is one of $(0 - 9)$. Assume that all licence plates are equally likely.
 - i. What is the probability that all symbols are different?
 - ii. What is the probability that all symbols are different and the first number is the largest among the numbers?

2. Cards

Shuffle a deck of 52 cards. What is the probability that

- (a) the top card is a heart?
- (b) all cards of the same suit end up next to each other?
- (c) the diamonds are together?

3. Committee

Four people are chosen from a group of ten persons consisting of four men and six women to serve on a committee. Three of the women are sisters. What is the probability that the four people chosen will

- (a) consists of four women?
- (b) consists of two men and two women?
- (c) consists of more women than men?
- (d) include the three sisters?

4. Tomatoes v. Caterpillars

Tomato plants emit distress signals when under attack. However, some caterpillars can use the chemicals in their saliva to 'silence' them.

At a research greenhouse, there are 24 tomato plants. Four plants are selected at random for testing without replacement. Suppose that six of the plants have been attacked by 'silencing' caterpillars whose saliva suppresses distress signals.

- (a) What is the probability that exactly one of the selected plants has been attacked by a silent caterpillar?
- (b) What is the probability that at least one of the selected plants has been attacked by a silent caterpillar?
- (c) In addition to the six plants attacked by silent caterpillars, four different plants have been infected by a fungus that prevents them from releasing distress signals altogether. What is the probability that exactly one selected plant has been attacked by a silent caterpillar, and exactly one selected plant has been infected by the fungus?

5. Passwords

A security system requires users to create a password that is **eight** characters long, and each character is one of: 26 lower case letters (a-z), 26 upper case letters (A-Z), and 10 digits (0-9).

- (a) What is the probability that a randomly selected password consists of only lower case letters (repetition allowed).
- (b) What is the probability that a randomly selected password starts with 4 letters, followed by 4 digits (repetition not allowed).
- (c) What is the probability that a randomly generated password contains only letters with the sequence 'HINT' in consecutive order, regardless of letter case? (repetition allowed).
- (d) What is the probability that a randomly generated password contains a mix of four digits and the sequence 'HINT' in consecutive order, regardless of letter case? (repetition not allowed).
- (e) How many passwords contain exactly one 'X' (in either upper or lower case) among the first three characters, assuming no repetition of characters is allowed?