Assignment 4

Instructions

- Deadline: **November 6 (4:00 pm)**

- Format: Hard copy (on paper) only. **Submissions by MIO will not be accepted.**

- Please show all of your work on your submission.

- Notation counts. Poor notation will result in a loss of marks.

- Please leave your answers as exact values. If using decimals, please report your answer to four decimal places.

- You are encouraged to ask your instructor for help, and/or discuss ideas with your classmates. However, you must produce fully explained individual solutions.

- Under no circumstances may you simply copy solutions obtained online or from a classmate.

- In unclear cases, you may be asked to explain your solutions in a Teams meeting, and your work may be refused altogether.

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1. **Meal Options**

   At a food court, customers must choose one of four main meal options: Burger, Chicken, Vegetarian, or Seafood. As a restaurant manager, you observe that: 40% of customers order a Burger; 25% order Chicken; 20% order a Vegetarian meal; and the rest order Seafood. Assuming that each customer makes their choice independently, what is the probability that for the next four customers:

   (a) All order a Burger?
   (b) All order Seafood or all order Chicken?
   (c) Each customer orders a different meal?
   (d) Exactly two order a Vegetarian meal?
   (e) All ordered Seafood, given that all ordered the same type of meal?

2. **Orange Juice**

A batch of 500 containers for frozen orange juice contains 25 that are defective. Two are selected, at random, with replacement from the batch.

(a) What is the probability that the second one selected is defective given that the first one was defective?

(b) What is the probability that both are defective?

(c) What is the probability that both are acceptable?

For parts (d) - (f), suppose that three containers are selected, at random, without replacement, from the batch.

(d) What is the probability that the third one selected is defective given that the first and second ones selected were defective?

(e) What is the probability that the third one selected is defective given that the first one selected was defective and the second one selected was okay?

(f) What is the probability that all three are defective?

3. **iPhones**

To avoid creeps from taking covert photos, iPhones bought in Japan (and seemingly also South Korea) cannot be muted when using the camera. The mandatory shutter sound was installed to reduce voyeuristic "up-skirt" photography - especially in crowded places like rush-hour trains.

The chips responsible for the camera's sound are produced at two manufacturing plants; one in Korea, and the other in Japan. The Korean factory makes 40% of the chips, while the Japanese plant produces the remaining 60%. Quality control has determined that 2% of the chips manufactured in Korea are defective, and 3% of the chips made in Japan are defective.

(a) If a randomly selected chip is found to be defective, what is the probability that it was manufactured in Korea?

(b) If a randomly selected chip is not defective, what is the probability that it was made in Japan?

(c) What is the probability that a randomly selected was made in Japan or not defective?

4. **Stock Prices**

A simplified model for the movement of the price of a stock supposes that on each day the stock's price either moves up 1 unit with probability p or moves down 1 unit with probability $1 - p$. The changes on different days are assumed to be independent.

(a) What is the probability that after 2 days the stock will be at its original price?

(b) Given that after 3 days the stock's price has increased by 1 unit, what is the probability that it went up on the first day?

(c) Are the events "the stock goes up on the first day" and "the stock's price increases after two days" independent? Justify your reasoning mathematically.

5. **Laptops**

Two customers visit a store and ask a salesperson for advice about laptops. The first customer decides to buy a laptop with probability 0.3, while the second customer buys one with probability 0.6. Their decisions are independent of each other.

Whenever a customer makes a purchase, the laptop is equally likely to be either Model A, which costs $\$ 1000$, or Model B, which costs $\$ 500$. These choices are also independent from one another and from whether the other customer buys.

(a) Determine the probability mass function of X.

(b) Determine the expected value of X, and interpret it in the context of the problem.

(c) Compute the variance and the standard deviation of X. Include units in your solution.