

Assignment 5

Instructions

- Deadline: **November 20 (4:00 pm)**
- Format: Hard copy (on paper) only. **Submissions by MIO will not be accepted.**
- Please show all of your work on your submission.
- Notation counts. Poor notation will result in a loss of marks.
- Please leave your answers as exact values. If using decimals, please report your answer to four decimal places.
- You are encouraged to ask your instructor for help, and/or discuss ideas with your classmates. However, you must produce fully explained individual solutions.
- Under no circumstances may you simply copy solutions obtained online or from a classmate.
- In unclear cases, you may be asked to explain your solutions in a Teams meeting, and your work may be refused altogether.

1. Bread Guilds

In the 14th century, bakers of white and brown bread split into two rival guilds and didn't reunite until the 17th century.

A recent study found that 45% of people prefer white bread, while the rest prefer brown bread. Suppose that 10 people are randomly selected.

- (a) What is the probability that two or three of them prefer white bread?
- (b) What is the probability that at least one person prefers white bread?
- (c) What is the probability that at fewer than four people prefer brown bread?
- (d) What is the expected number of people in the group that prefer brown bread? What is the standard deviation?

2. Gut Biome

Not to be tried at home, but doctors in Finland believe that feeding babies small amounts of their mother's poo may strengthen their gut bacteria.

In a pilot study, researchers measured the time (in hours) it took for infants' gut microbiomes to reach a "balanced" state after the treatment. Suppose this time follows a normal distribution with a mean of 72 hours and a standard deviation of 10 hours.

- (a) What is the probability that a randomly selected baby achieves a balanced gut microbiome in less than 60 hours?
- (b) What is the probability that the balance time is more than 85 hours?
- (c) What is the probability that the balance time is between 70 and 80 hours?
- (d) Only the slowest 10% of babies take this long to adjust. What is the minimum number of hours required to be in this slowest 10% group?

3. Thisbe and Pyramus

In Greek mythology, Pyramus and Thisbe were lovers separated by a wall. Today, the Pyramus and Thisbe Society is a group of lawyers who specialise in wall-related disputes.

A recent report revealed that the probability a randomly selected lawyer in Greece belongs to this society is 0.40.

- (a) In a small firm of 6 lawyers, what is the probability that 4 or 5 of them belong to the Pyramus and Thisbe Society?
- (b) In a larger regional sample of 80 lawyers, what is the probability that at least 20 are members of the society?
- (c) In a nationwide survey of 300 lawyers, estimate the probability that between 110 and 135 (exclusive) are members.
- (d) In a separate study of 400 lawyers, estimate the probability that fewer than 270 are not members of the society.

4. Goldfish

In 2023, a Japanese YouTuber used motion tracking so his pet fish could remotely control a Nintendo Switch console and ‘play’ video games. Left alone for seven hours, the fish logged into the Nintendo store, set up a PayPal account and deposited 500 yen from his credit card.

Researchers later became curious about the swimming patterns of such “tech-savvy” goldfish. They found that the distance (in cm) a goldfish swims in a 5-minute period follows a normal distribution with a mean of 45 cm and a standard deviation of 6 cm.

- (a) What is the probability that a randomly selected goldfish swims more than 55 cm in 5 minutes?
- (b) Suppose a goldfish has already surpassed 50 cm. What is the probability that it ends up swimming between 60 and 65 cm?
- (c) How far must a goldfish swim to be in the top 5% of swimmers?
- (d) In a tank of 120 goldfish, each one attempts to “press a button” using motion tracking. Suppose the probability of success for a single fish is 0.12. What is the probability that fewer than 10 of them succeed?
- (e) In a smaller experiment with 10 goldfish, what is the probability that exactly 1 or 2 swim more than 55 cm in a 5-minute trial?

5. Security at the Louvre

In 1911, a journalist crept into a sarcophagus in the Louvre and spent the night there to show how bad the museum’s security was.

Modern researchers studying museum security incidents found that the response time (in minutes) to detect a security breach follows a normal distribution with a mean of 42 minutes and a standard deviation of 7 minutes.

- (a) What is the probability that a randomly selected incident goes undetected for more than 53 minutes?
- (b) If a breach has already gone undetected for at least 40 minutes, what is the probability that it lasts longer than 55 minutes?
- (c) Find the detection time range that captures the middle 99% of incidents.
- (d) In a review of 120 simulated break-ins, each one is caught by a motion sensor with probability 0.93. What is the probability that fewer than 110 are detected?
- (e) In a smaller trial of 10 museum guards, each guard independently notices suspicious activity with probability 0.8. What is the probability that exactly 14 or 16 of them spot the intruder?