L13. Confidence Interval on the Mean; Single Population, Variance Known

Example 1

Past experience has indicated that the breaking strength of yarn used in manufacturing drapery material is normally distributed and that σ = 2 psi. A random sample of nine specimens is tested, and the average breaking strength is found to be 98 psi.

Find a 95% two-sided confidence interval on the true mean breaking strength.

Solution

Example 2

The diameter of a cable is known to have a normal distribution with $\sigma = 0.04$. A random sample of 25 cables was taken, and the average diameter for this sample was found to be 1.50 inches.

a. Find a 80% two-sided confidence interval for the mean diameter.

b. Find a 90% two-sided confidence interval on the mean diameter of the cable.

c. Find a 99% two-sided confidence interval on the mean diameter.

d. Which of the above intervals is the longest?

e. Which of the above intervals is the most precise?

Solution

Example 3

The lifetime in hours of a 75-watt light bulb is known to be normally distributed with $\sigma = 25$ hours. A random sample of 20 bulbs has a mean life of $\overline{x} = 1014$ hours.

a. Construct a 95% two-sided confidence interval on the mean life.

b. Suppose that we want the error for estimating the mean life to be at most five hours at 95% confidence. What sample size should be used?

Solution

Example 4

A bank manager wants to know the mean amount paid per month by home owners living in the town of Saint-Augustin in Mirabel. A random sample of 50 residents selected from the area showed that they pay an average of \$1575 per month for their mortgages. The population standard deviation for such mortgages is \$215.

a. Find a 97% confidence interval for the mean amount paid per month by all home owners in this area.

b. How large of a sample should be selected so that the estimate for the average monthly mortgage payment is within \$50 of the true population mean with 97% confidence?

c. How large of a sample should be selected so that the estimate for the average monthly mortgage payment is within \$50 of the true population mean with 95% confidence?

Solution

Example 5

Suppose that for a particular brand of concrete, the compressive strength of the material is known to be normally distributed with a variance of 6.25 psi. In a quality control test, 15 samples were taken, and their mean compressive strength was found to be 54.41 psi.

a. Determine a two-sided 98% confidence interval for the average compressive strength of the concrete.

b. Determine a 98% lower confidence bound for the true average compressive strength of the concrete.

Solution

Example 6

A city planner wants to estimate the average monthly water usage in the city of Montreal. He selects a random sample of 40 households, which gave a mean water usage to be 3415.70 gallons over a 1-month period. Suppose that it is known that water usage in Montreal follows a normal distribution with a standard deviation is 389.60 gallons.

a. Make an 85% confidence interval for the average monthly water usage for all households in Montreal.

b. What sample size is needed so that the estimate for the average monthly water usage in this city is within 60 gallons of the actual population mean with 85% confidence?

c. Construct an 85% upper-confidence bound for the mean water usage for all households in Montreal.

Solution