L14. Confidence Interval on the Mean; Single Population, Variance Unknown

Example 1

Find the value of the t for the $t-$value for each of the following.

a. Area in the right tail = 0.05, and df = 12

b. Area in the right tail = 0.125, and df = 58

c. Area in the left tail = 0.005, and df = 20

d. Area in the left tail = 0.01, and df = 1500

e. Area in the right tail = 0.05, for a sample size n = 25

f. Area in the left tail = 0.025, for a sample of size n = 15

Solution

a. $t_{0.05,12} = 1.782$

b. $t_{0.125,58} = 1.164$

c. $t_{0.005,20} = -2.845$

d. $t_{0.01,1500} = -2.326$

e. $t_{0.05,25} = 1.708$

f. $t_{0.025,15} = -2.131$

Example 2

A random sample of 16 airline passengers at Trudeau airport showed that the mean time spent waiting in line to check in at the ticket counter was 31 minutes with a standard deviation of 7 minutes. Assuming that wait times for all passengers are normally distributed.

Construct a 90% confidence interval for the mean time spent waiting in line by all passengers at this airport.

Solution

We have that: $\overline{x} = 31 \qquad s = 7 \qquad n = 16$

90% confidence $\quad\Rightarrow\quad t_{0.05,15} = 1.753$

Confidence interval:

$$\overline{x} \pm t_{\alpha/2,n-1} \cdot \frac{s}{\sqrt{n}} \quad\Rightarrow\quad 31 \pm 1.753 \cdot \frac{7}{\sqrt{16}}$$

$$31 \pm 3.0678 \quad\Rightarrow\quad 27.9322 < \mu < 34.0678$$

Interpretation:

With repeated sampling, we are 90% confident that the actual time spent waiting at the ticket counter is between 27.9322 and 34.0678 minutes.

Example 3

Almost all employees working for financial companies in New York receive large bonuses at the end of the year. A sample of 65 employees selected from financial companies in New York City showed that they received an average bonus of \$55 000 last year with a standard deviation of \$18 000.

a. Construct a 95% confidence interval for the average bonus that all employees working for financial companies in New York receive last year.

b. Construct a 98% confidence interval for the average bonus that all employees working for financial companies in New York receive last year.

c. Construct a 99% confidence interval for the average bonus that all employees working for financial companies in New York receive last year.

d. Of these three intervals, which is the least precise?

Solution

We are given: $\overline{x} = 55000 \quad s = 18000 \quad n = 65$

a. 95% confidence $\quad\Rightarrow\quad t_{0.025,64} = 2.00$

$$\overline{x} \pm t_{\alpha/2,n-1} \cdot \frac{s}{\sqrt{n}}$$

$$55000 \pm 2.00 \cdot \frac{18000}{\sqrt{65}}$$

$$55000 \pm 4465.25 \quad\Rightarrow\quad 50534.74955 < \mu < 59465.25045$$

Interpretation:

With repeated sampling, we are 95% confident that the actual average salary of employees working in New York financial companies is between \$50534.45 and \$59465.25.

b. 98% confidence $\Rightarrow$ $t_{0.01,64} = 2.39$

$$\overline{x} \pm t_{\alpha/2,n-1} \cdot \frac{s}{\sqrt{n}}$$
$$55000 \pm 2.39 \cdot \frac{18000}{\sqrt{65}}$$
$$55000 \pm 5335.9743 \qquad \Rightarrow \qquad 49664.0257 < \mu < 60335.9743$$

Interpretation:

With repeated sampling, we are 98% confident that the actual average salary of employees working in New York financial companies is between \$49664.03 and \$60335.97.

c. 99% confidence $\Rightarrow$ $t_{0.005,64} = 2.66$

$$\overline{x} \pm t_{\alpha/2,n-1} \cdot \frac{s}{\sqrt{n}}$$
$$55000 \pm 2.66 \cdot \frac{18000}{\sqrt{65}}$$
$$55000 \pm 5938.7831 \qquad \Rightarrow \qquad 49061.2169 < \mu < 60938.7831$$

Interpretation:

With repeated sampling, we are 99% confident that the actual average salary of employees working in New York financial companies is between \$49061.22 and \$60938.78.

d. The least precise of the three confidence intervals, is the the 99% confidence interval.

Example 4

A company randomly selected nine office employees and secretly monitored their computers for one month. The time (in hours) spent by these employees used their computers for non-job related activities during this month are as follows

$$7 \quad 12 \quad 9 \quad 8 \quad 11 \quad 4 \quad 14 \quad 1 \quad 6$$

Assuming that such times are normally distributed,

a. Calculate the sample average, $\overline{x}$, and sample standard deviation, s

b. Calculate a 95% lower-confidence bound to the corresponding mean for all employees of this company.

c. Calculate a 90% upper-confidence bound to the corresponding mean for all employees of this company.

Solution

a. $\overline{x} = \frac{1}{n}\sum_{i=1}^{9} x_i = \dfrac{7+12+9+\cdots+6}{9} = 8$

$s^2 = \frac{1}{n-1}\sum_{i=1}^{9}(x_i - \overline{x})^2 = \dfrac{(7-8)^2+(12-8)^2+\cdots+(6-8)^2}{9-1} = 16.5$

$s = \sqrt{16.5}$

b. 95% lower confidence bound $\Rightarrow$ $t_{0.05,8} = 1.860$

$$\overline{x} - t_{\alpha,n-1} \cdot \frac{s}{\sqrt{n}} \leq \mu$$

$$8 - 1.860 \cdot \frac{\sqrt{16.5}}{\sqrt{9}} \leq \mu \quad \Rightarrow \quad 5.4815 \leq \mu$$

Interpretation:

With repeated sampling we are 95% confident that the average number of hours that employees spent on their computers doing non-job related tasks is at least 5.4815 hours per month.

c. 90% upper confidence bound $\Rightarrow$ $t_{0.10,8} = 1.397$

$$\mu \leq \overline{x} + t_{\alpha,n-1} \cdot \frac{s}{\sqrt{n}}$$

$$\mu \leq 8 + 1.397 \cdot \frac{\sqrt{16.5}}{\sqrt{9}} \quad \Rightarrow \quad \mu \leq 9.8915$$

Interpretation:

With repeated sampling we are 90% confident that the average number of hours that employees spent on their computers doing non-job related tasks is at most 9.8915 hours per month.