

Class Exercise 10

1. Flight Control Avionics

An autonomous aerial vehicle uses an onboard navigation unit equipped with $n = 5$ independent sensor channels. During cruise flight, each channel operates reliably with probability $p = 0.80$, independently of the others. The flight computer maintains stable control if **at least 3** channels operate reliably.

- a. Let X be the number of reliably operating channels. State the distribution of X , and compute its expected value $E[X]$, variance $\text{Var}(X)$, and standard deviation σ_X .
- b. Compute the probability that the flight computer maintains stable control during cruise flight.
- c. In-flight telemetry confirms that the aircraft is maintaining stable control ($X \geq 3$).
 - (i) Given that the flight control is stable, what is the conditional probability that **all 5** channels are operating reliably?
 - (ii) Given that the flight control is stable, what is the conditional probability that **exactly 3** channels are operating?
 - (iii) Given that at least one channel has failed ($X \leq 4$), what is the conditional probability that the flight control is still stable ($X \geq 3$)?
- d. In a specialized high-altitude flight mode, an individual channel operates reliably only if two sequential stages succeed: the sensor hardware remains within safe operating temperatures (probability 0.95), and the signal processor successfully filters atmospheric noise (conditional probability 0.90 given safe temperatures). If the temperature is unsafe, the channel fails.
 - (i) Find the probability p_{new} that an individual channel operates reliably in this mode.
 - (ii) In a backup module with $n = 4$ such high-altitude channels, what is the probability that **at least one** channel fails?

2. Server Anomaly Detection

A network operations center monitors an enterprise database server. Minor system anomalies (such as packet retransmissions and query timeouts) occur according to a Poisson process. The server operates in one of two states:

- **Normal State (N):** Occurs 90% of the time ($P(N) = 0.90$). Anomalies occur at an average rate of $r_N = 0.5$ anomalies per hour.
 - **Degraded State (D):** Occurs 10% of the time ($P(D) = 0.10$). Anomalies occur at an elevated average rate of $r_D = 2.0$ anomalies per hour.
- a. An automated monitoring system logs anomalies over a **2-hour window** ($t = 2$ hours).
 - (i) Rescale the Poisson rates to find the parameter (expected number of anomalies in 2 hours) under the Normal state (λ_N) and under the Degraded state (λ_D).
 - (ii) Find the probability of observing **no anomalies** in this 2-hour window under the Normal state, and under the Degraded state.
 - b. During a specific 2-hour window, the monitor records exactly **4 anomalies**.
 - (i) Compute the likelihood $P(X = 4 \mid N)$ of observing 4 anomalies under the Normal state.
 - (ii) Compute the likelihood $P(X = 4 \mid D)$ of observing 4 anomalies under the Degraded state.
 - c. Using the Law of Total Probability and Bayes' Rule:
 - (i) Find the overall probability $P(X = 4)$ of recording 4 anomalies in a 2-hour window.
 - (ii) Compute the posterior probability $P(D \mid X = 4)$ that the server is currently operating in the Degraded state given that 4 anomalies were observed. Compare this result to the prior probability $P(D) = 0.10$.
 - d. Following this alert, a systems engineer initiates an immediate **30-minute** ($t = 0.5$ hours) diagnostic test. Assuming the server is indeed in the Degraded state ($r_D = 2.0$ per hour):
 - (i) Rescale the rate to find the expected number of anomalies λ_{test} during this 30-minute test.
 - (ii) What is the probability that the engineer observes **at least one** anomaly during the 30-minute test?