Class Exercise 2 — Solutions

1. **Trimodal Distribution**

   Describe a realistic scenario that would produce a trimodal histogram. Identify the continuous variable being measured and explain why three distinct peaks occur.

Sample Solution:

1. Realistic Scenario: Customer arrival times at a popular, all-day diner or casual restaurant located in a bustling business and commercial district over a typical 24-hour weekday cycle.

2. Continuous Variable Being Measured: Customer arrival time T, measured continuously in hours from midnight (e.g., $T = 8.25$ corresponds to 8:15 AM).

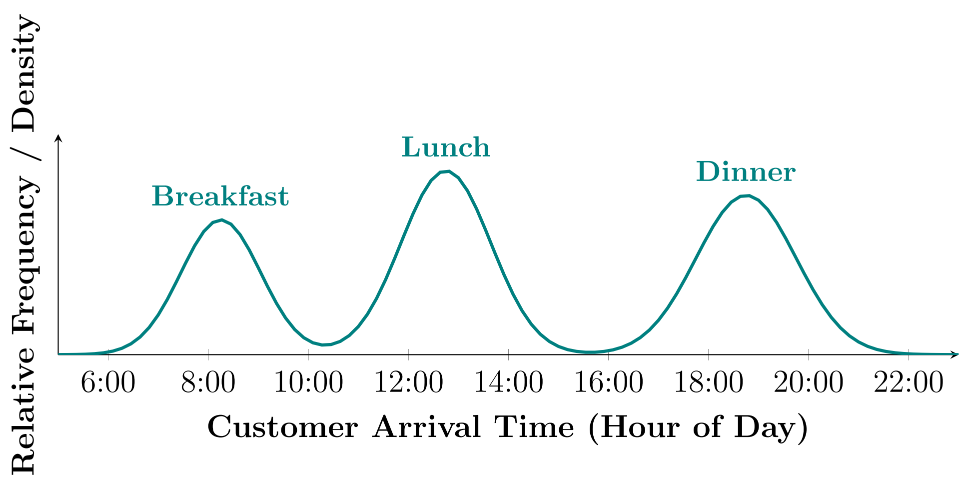
3. Explanation of the Three Distinct Peaks (Modes): The histogram of arrival times across the day will display three distinct peaks corresponding to the three standard daily mealtimes:

(a) **Peak 1 (Breakfast Rush, ≈ 7:30 AM – 9:00 AM):** Commuters and workers purchasing coffee and breakfast before the workday starts.

(b) **Peak 2 (Lunch Rush, ≈ 12:00 PM – 1:30 PM):** Employees from nearby businesses taking their midday meal break.

(c) **Peak 3 (Dinner Rush, ≈ 6:00 PM – 7:30 PM):** Individuals and families dining after finishing work shifts.

Between these peak mealtimes (mid-morning, mid-afternoon, and late night), customer arrivals drop considerably. Because the overall distribution blends three separate surges of customer demand, the histogram is **trimodal**.



Alternative valid scenarios include: (i) vehicle speeds on a highway across morning rush hour, midday lull, and nighttime free-flow; (ii) hourly electricity demand on a power grid; or (iii) body mass in a mixed sample of three distinct penguin species.

2. **AI Vision Quality Inspection**

   Last month, a robotics startup trained an AI vision system to inspect assembled consumer electronics moving along a factory conveyor belt. During an early trial run, the AI repeatedly misidentified human fingers as "defective hotdogs" and triggered automated emergency halts. Quality control engineers recorded the number of false emergency halts triggered per production shift across several manufacturing lines. The results are summarized in the table below:

Number of Shifts	Number of False Halts
15	1
28	2
34	3
27	4
16	5

Calculate the mean and the median number of false emergency halts per shift.

Solution:

Step 1: Distinguish the Variable from the Frequency

- We are asked for the **mean and median number of false emergency halts per shift**.
- The observation unit is a **shift**, so the **variable of interest** is: $x =$ Number of False Halts $\in \{1, 2, 3, 4, 5\}$.
- The number of shifts is the **frequency** f with which that count was observed.
- **Common Pitfall to Avoid:** Frequency f appears in the *first* column and variable x in the *second* column. Both are integer counts. Do **not** compute statistics on the frequency column $\{15, 16, 27, 28, 34\}$. The frequency indicates how many shifts experienced each halt count.

Step 2: Calculate the Mean ($\bar{x}$)

The total number of shifts surveyed is $N = \sum f = 15 + 28 + 34 + 27 + 16 = 120$. We compute the weighted sum $\sum(x \cdot f)$:

$$\sum(x \cdot f) = 1(15) + 2(28) + 3(34) + 4(27) + 5(16) = 15 + 56 + 102 + 108 + 80 = 361 \text{ halts.}$$

The sample mean is:

$$\bar{x} = \frac{\sum(x \cdot f)}{N} = \frac{361}{120} \approx \mathbf{3.01} \text{ false emergency halts per shift} \quad (\text{or } 3.008).$$

Step 3: Calculate the Median

Since $N = 120$ is even, the median lies halfway between the 60^{th} and 61^{st} ordered observations:

$$\text{Median position} = \frac{x_{(60)} + x_{(61)}}{2}.$$

We construct the frequency and cumulative frequency table sorted in ascending order of x:

False Halts (x)	Shifts (f)	Product ($x \cdot f$)	Cumulative Freq. (cf)	Ordered Positions
1	15	15	15	$1^{\text{st}} - 15^{\text{th}}$
2	28	56	43	$16^{\text{th}} - 43^{\text{rd}}$
3	**34**	**102**	**77**	$\mathbf{44^{\text{th}} - 77^{\text{th}}}$
4	27	108	104	$78^{\text{th}} - 104^{\text{th}}$
5	16	80	120	$105^{\text{th}} - 120^{\text{th}}$
Total	$N = 120$	$\sum(x \cdot f) = 361$		

Both the 60^{th} and 61^{st} observations fall in the category $x = 3$ (positions 44–77). Thus, $x_{(60)} = 3$ and $x_{(61)} = 3$:

$$\text{Median} = \frac{3 + 3}{2} = \mathbf{3} \text{ false emergency halts per shift.}$$

3. **Tensile Strength of Titanium Alloy Brackets**

   A quality control engineer at an aerospace manufacturing facility tested the tensile strength of six newly fabricated titanium alloy brackets designed for an aircraft wing assembly. The six specimens were loaded in a universal testing machine until mechanical failure. The recorded failure loads (in kilonewtons, kN) are:

   $$40, \quad 41, \quad 44, \quad 47, \quad 48, \quad 50$$

   (a) Calculate the sample mean failure load, $\bar{x}$ (include appropriate units).

   (b) Compute the sample variance s^2 of the failure loads. Show your work clearly by constructing a table with deviations $(x_i - \bar{x})$ and squared deviations $(x_i - \bar{x})^2$.

   (c) Compute the sample standard deviation s of the failure loads (include appropriate units).

Solution:

(a) Sample Mean Failure Load $\bar{x}$:

$$\sum_{i=1}^{6} x_i = 40 + 41 + 44 + 47 + 48 + 50 = 270 \text{ kN}$$

With sample size $n = 6$:

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{270}{6} = \mathbf{45} \text{ kN}$$

(b) Sample Variance s^2:

We construct the table of deviations and squared deviations from the sample mean $\bar{x} = 45$ kN:

Specimen i	**Failure Load x_i (kN)**	**Deviation $(x_i - \bar{x})$**	**Squared Dev. $(x_i - \bar{x})^2$**
1	40	$40 - 45 = -5$	$(-5)^2 = 25$
2	41	$41 - 45 = -4$	$(-4)^2 = 16$
3	44	$44 - 45 = -1$	$(-1)^2 = 1$
4	47	$47 - 45 = +2$	$(2)^2 = 4$
5	48	$48 - 45 = +3$	$(3)^2 = 9$
6	50	$50 - 45 = +5$	$(5)^2 = 25$
Total	$\sum x_i = 270$ kN	$\sum(x_i - \bar{x}) = 0$	$\sum(\mathbf{x_i} - \mathbf{\bar{x}})^\mathbf{2} = \mathbf{80}$ $\text{kN}^\mathbf{2}$

Check: The sum of deviations is $(-5) + (-4) + (-1) + 2 + 3 + 5 = 0$, confirming our mean.

Using the sample variance formula with degrees of freedom $n - 1 = 6 - 1 = 5$:

$$s^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1} = \frac{80}{6-1} = \frac{80}{5} = \mathbf{16} \text{ kN}^\mathbf{2}$$

Shortcut Formula Check: $s^2 = \frac{\sum x_i^2 - \frac{(\sum x_i)^2}{n}}{n-1} = \frac{12230 - \frac{270^2}{6}}{5} = \frac{12230 - 12150}{5} = \frac{80}{5} = 16 \text{ kN}^2$.

(c) Sample Standard Deviation s:

$$s = \sqrt{s^2} = \sqrt{16 \text{ kN}^2} = \mathbf{4} \text{ kN}$$

Note on Degrees of Freedom: Because these six brackets represent a sample from an ongoing manufacturing process, division by $n - 1 = 5$ gives an unbiased estimate of the process variance. If treated strictly as a closed population of 6 items, the population variance would be $\sigma^2 = \frac{80}{6} \approx 13.33 \text{ kN}^2$ and $\sigma \approx 3.65$ kN.