Class Exercise 3

1. **Chebyshev's Inequality vs. The Empirical Rule**

   A manufacturer of avionics equipment tests the operational lifetime of an autonomous drone navigation sensor before recalibration is required. The sensor lifetime has a mean of $\mu = 600$ hours and a standard deviation of $\sigma = 40$ hours.

   (a) Suppose nothing is known about the shape of the distribution of sensor lifetimes.
      (i) What interval is guaranteed by Chebyshev's inequality to contain at least 75% of the sensor lifetimes?
      (ii) What minimum percentage of sensors will have lifetimes between 480 hours and 720 hours?

   (b) Now suppose extensive historical quality records confirm that sensor lifetimes are **bell-shaped and symmetric**.
      (i) According to the Empirical Rule, approximately what percentage of sensors will have lifetimes between 520 hours and 680 hours?
      (ii) Approximately what percentage of sensors will have lifetimes between 480 hours and 720 hours?

   (c) A junior technician remarks: *"Chebyshev's inequality guarantees at least 75% of sensors fall between 520 and 680 hours, whereas the Empirical Rule states that about 95% fall in that same interval. Since 75% and 95% disagree, one of the two rules must be flawed."* Briefly explain why the technician's conclusion is incorrect.

2. **Standard Deviation from a Frequency Table**

   A telecommunications engineer monitored the number of transmission packet errors observed during 10-minute testing windows across a satellite data downlink. A random sample of $n = 25$ testing windows yielded the following frequency distribution:

Transmission Errors (x)	**Frequency / Number of Windows (f)**
0	4
1	8
2	1
3	8
4	4

   (a) Calculate the sample mean $\bar{x}$ number of transmission errors per window.

   (b) Construct a computational table showing columns for x, f, fx, and fx^2, and use it to compute the sample variance s^2.

   (c) Determine the sample standard deviation s (round to four decimal places).

   (d) A student attempts to calculate the variance by taking the five values $\{0, 1, 2, 3, 4\}$ and calculating their sample variance as $s^2 = 2.5$. Identify the conceptual error in this approach.

3. **Effects of Shifting and Scaling on Spread**

A civil engineering laboratory measures the daily thermal expansion of a set of steel bridge joints during summer temperature swings. The measured expansions x (in millimetres) have a sample mean of $\bar{x} = 15$ mm, a sample variance of $s^2 = 9$ mm^2, and a sample standard deviation of $s = 3$ mm.

(a) **Shifting (Sensor Calibration):** Due to a calibration error, each sensor had an initial reading of $+4$ mm before testing began. To correct for this zero-offset, the engineers subtract 4 mm from every observation, defining the corrected data as:

$$y_i = x_i - 4.$$

Find the new mean $\bar{y}$, the new variance s_y^2, and the new standard deviation s_y. Explain conceptually why the standard deviation does not change when a constant is added or subtracted.

(b) **Scaling (Unit Conversion):** The team needs to report the original measurements in centimetres rather than millimetres. The conversion formula divides each measurement by 10 (or multiplies by 0.1):

$$u_i = 0.1\, x_i.$$

Find the new mean $\bar{u}$, the new variance s_u^2, and the new standard deviation s_u (include appropriate units with each).

(c) **Combined Linear Transformation with Negative Sign:** In an automated structural health algorithm, a normalized risk index z_i is computed from each original expansion measurement x_i using the formula:

$$z_i = 100 - 4\, x_i.$$

Calculate the mean $\bar{z}$, the variance s_z^2, and the standard deviation s_z of the risk index. (*Caution: Pay careful attention to the sign of the standard deviation!*)