

Class Exercise 6

1. **Agentic swarm**

Masterbot manages 22 AI agents residing in my laptop. There are four distinct group projects, which need 9, 5, 4 and 4 agents respectively. Every agent is assigned to exactly one project.

- In how many ways can the 22 agents be assigned to the four projects?
- Three particular agents refuse to work with one another, so no two of these three may be on the same team. In how many ways can the assignment be done now?

2. **Poker hand**

A poker hand consists of 5 cards dealt from a standard 52-card deck. A hand is **Two Pairs** when it contains two cards of one rank, two cards of a second, different rank, and a fifth card whose rank differs from both. For example: $5\heartsuit, 5\spadesuit, Q\clubsuit, Q\spadesuit, 10\diamondsuit$. What is the probability that a randomly dealt poker hand is Two Pairs?

3. **Koala thieves**

In 2006, thieves planning to steal a koala from a zoo in Australia had to change their minds when it proved too vicious to abduct. The perpetrators later confessed that their original plan was to swap the koala for drugs, but after it scratched the hell out of them, they stole a 1.4-metre crocodile instead, which they claimed was a lot easier.

- A witness saw a car speeding away from the zoo. Its licence plate had 7 symbols: a letter (V or W), then the three digits 4, 7, 8 in some order, then the three letters A, C, E in some order. The witness could not remember the order of the digits or of the last three letters. How many plates would need to be checked to be sure of including the suspect car?
- In other parts of Australia, a licence plate has seven symbols in the pattern

digit, letter, letter, letter, digit, digit, digit,

where a letter is any of the 26 letters A–Z and a digit is any of 0–9. Assume all such plates are equally likely.

- What is the probability that all seven symbols are different?
- What is the probability that all seven symbols are different *and* the first digit (the leftmost symbol) is the largest of the four digits?

4. **Cards**

A full deck of 52 cards is shuffled so that all $52!$ orderings are equally likely.

- What is the probability that the four Aces are in four consecutive positions in the deck? (You do not have to convert the answer to a decimal.)
- What is the probability that the $A\clubsuit$ is not immediately above the $K\clubsuit$ **and** the $A\spadesuit$ is not immediately above the $K\spadesuit$?