Class Exercise

1. **Rebel Magpies**

In a 2022 pilot study, scientists fitted 5 Australian magpies with tiny GPS trackers that could only be removed using a magnet or scissors. Within 10 minutes, a dominant female had successfully removed the tracker from a younger bird, and within 3 days all of the devices had been removed. In a larger follow-up study, a wildlife ecologist fits a random sample of 40 magpies with the same type of tracker and records how long (in hours) each device stays on before being removed by the bird or a flock-mate. The sample yields an average attachment time of $\bar{x} = 20.0$ hours. From previous deployments, the population standard deviation of attachment times is known to be 4.0 hours.

(a) What are the underlying assumptions for creating a 95% z-score interval?

Solution:

- **Random sample:** The sample of 40 magpies was chosen randomly.
- **Normality:** The distribution of the sample mean should be approximately normal.

(b) Construct and interpret a 95% z-score confidence interval for the true mean attachment time of the trackers on Australian magpies.

Solution:

Given $n = 40$, $\bar{x} = 20.0$, and $\sigma = 4.0$. For a 95% confidence level, the critical value is $z_{\alpha/2} = 1.96$.

Margin of Error $(E) = 1.96\left(\frac{4.0}{\sqrt{40}}\right) \approx 1.24$

Confidence Interval $= 20.0 \pm 1.24 = [18.76, 21.24]$

Interpretation: We are 95% confident that the true mean attachment time of the trackers on Australian magpies is between 18.76 hours and 21.24 hours.

(c) A technology company that manufactures the trackers claims that the devices stay attached for *at least* 22 hours on average when used with magpies. Based on the 95% confidence interval you constructed, can this claim be validated? Justify your answer.

Solution:

No, the claim cannot be validated. The entire 95% confidence interval ($[18.76, 21.24]$ hours) lies completely below 22 hours. This provides strong statistical evidence that the true average attachment time is less than the 22 hours claimed by the company.

(d) If the ecologist wants the margin of error for a 95% confidence interval to be no more than 0.75 hours, what is the minimum sample size needed?

Solution:

We want $E \leq 0.75$. Using the sample size formula for a mean:

$$n = \left(\frac{z_{\alpha/2}\sigma}{E}\right)^2 = \left(\frac{1.96(4.0)}{0.75}\right)^2 = \left(\frac{7.84}{0.75}\right)^2 \approx (10.453)^2 \approx 109.27$$

Since we cannot sample a fraction of a bird, we always round up to the next whole integer. The minimum sample size needed is $n = 110$.

2. **ChatGPT Energy Use**

At a clean-tech research lab, an engineer collects data on the electricity required (in watt-hours) for each of 12 randomly selected Claude queries under controlled conditions. The sample of 12 queries yields a mean electricity consumption of 8.2 watt-hours per query, with a sample standard deviation of 2.5 watt-hours.

(a) What are the underlying assumptions for creating a 95% t-score interval?

Solution:

- **Random sample:** The 12 queries were selected randomly.
- **Normality:** Because the sample size is small ($n = 12 < 30$), we must assume that the population of electricity consumption for Claude queries follows an approximately normal distribution.

(b) Construct and interpret a 95% t-score confidence interval for the true mean electricity consumption of a Claude query.

Solution:
Given $n = 12$, $\bar{x} = 8.2$, and $s = 2.5$. The degrees of freedom is $df = 12 - 1 = 11$. For a 95% confidence level, $t_{\alpha/2} = 2.201$.
Margin of Error $(E) = 2.201\left(\frac{2.5}{\sqrt{12}}\right) \approx 1.59$
Confidence Interval $= 8.2 \pm 1.59 = [6.61, 9.79]$
Interpretation: We are 95% confident that the true mean electricity consumption of a Claude query is between 6.61 and 9.79 watt-hours.

(c) A "typical" Google search uses about 0.8 watt-hours of electricity. Using your 95% confidence lower bound, does the data provide evidence that a Claude query uses more energy on average than a Google search? Explain.

Solution:
Yes, the data provides strong evidence. The lower bound of our 95% confidence interval is 6.61 watt-hours. Because this lower bound is well above the 0.8 watt-hours used by a Google search, we are highly confident that Claude queries use significantly more energy on average.

(d) Construct and interpret a 98% t-score confidence upper bound for the true mean electricity consumption of a Claude query.

Solution:
For a 98% upper confidence bound, all of the α error goes into the right tail, meaning $\alpha = 0.02$. With $df = 11$, the critical t-value is $t_{0.02} \approx 2.491$.
Margin of Error $(E) = 2.491\left(\frac{2.5}{\sqrt{12}}\right) \approx 1.80$
Upper Bound $= \bar{x} + E = 8.2 + 1.80 = 10.00$
Interpretation: We are 98% confident that the true mean electricity consumption of a Claude query is at most 10.00 watt-hours.