

Class Exercise

1. Lavaland

Rejean is the drinking water inspector for the city of Lavaland. Historical records show that the arsenic concentration in the drinking water coming from a specific lake is 0.12 ppm with standard deviation 0.05 ppm and the distribution is normal. On a recent day the measurement of the arsenic concentration in drinking water taken from this lake exceeds the safety level of 0.25 ppm. What is the probability that the arsenic concentration will exceed 0.25 ppm on a random day? Is this event statistically significant?

Solution:

Let X be the arsenic concentration, $X \sim N(0.12, 0.05^2)$.

We want to find the probability $P(X > 0.25)$.

$$z = \frac{0.25 - 0.12}{0.05} = \frac{0.13}{0.05} = 2.6$$

$$P(Z > 2.6) = 1 - P(Z \leq 2.6) = 1 - 0.9953 = 0.0047$$

The probability of exceeding 0.25 ppm is approximately 0.47%.

Yes, this event is statistically significant because the probability of it occurring by chance is very low, less than the standard threshold of 5% (i.e., p -value = 0.0047 < 0.05).

2. Woodpeckers

The hairy woodpeckers, endemic to North America, have normally distributed weights (as adults) with mean $\mu = 25g$ and standard deviation of $\sigma = 2g$. What proportion of hairy woodpeckers:

- Have weights exceeding 35g?
- Have weights below 21g?
- Have weights in the range 21 – 28g?

What weight corresponds to:

- The bottom 10% of the distribution?
- The top 10% of the distribution?

Solution:

Let X be the weight of a hairy woodpecker, $X \sim N(25, 2^2)$.

a) $P(X > 35)$:

$$z = \frac{35 - 25}{2} = 5 \implies P(Z > 5) \approx 0 \text{ (Virtually 0\%)}$$

b) $P(X < 21)$:

$$z = \frac{21 - 25}{2} = -2 \implies P(Z < -2) = 0.0228 \text{ (2.28\%)}$$

c) $P(21 < X < 28)$:

$$z_1 = -2, \quad z_2 = \frac{28 - 25}{2} = 1.5$$

$$P(-2 < Z < 1.5) = P(Z < 1.5) - P(Z < -2) = 0.9332 - 0.0228 = 0.9104 \text{ (91.04\%)}$$

d) Bottom 10% ($P(X < x) = 0.10$):

$$z \approx -1.28 \implies x = \mu + z\sigma = 25 + (-1.28)(2) = 25 - 2.56 = 22.44 \text{ g}$$

e) Top 10% ($P(X > x) = 0.10 \implies P(X \leq x) = 0.90$):

$$z \approx 1.28 \implies x = 25 + (1.28)(2) = 25 + 2.56 = 27.56 \text{ g}$$

3. Merlins

The merlin is a species of small falcons present in Quebec only during the summers. The wingspans of merlins are normally distributed with mean $\mu = 62\text{cm}$ and standard deviation of $\sigma = 5\text{cm}$. Compute the values for the 20th, 40th, 60th and 80th percentile in the distribution of merlin wingspans.

Solution:

Let X be the wingspan, $X \sim N(62, 5^2)$. We need to find the x values corresponding to cumulative probabilities of 0.20, 0.40, 0.60, and 0.80.

- 20th percentile ($P(Z < z) = 0.20$): $z \approx -0.84 \implies x = 62 + (-0.84)(5) = 62 - 4.2 = 57.8 \text{ cm}$.
- 40th percentile ($P(Z < z) = 0.40$): $z \approx -0.25 \implies x = 62 + (-0.25)(5) = 62 - 1.25 = 60.75 \text{ cm}$.
- 60th percentile ($P(Z < z) = 0.60$): $z \approx 0.25 \implies x = 62 + (0.25)(5) = 62 + 1.25 = 63.25 \text{ cm}$.
- 80th percentile ($P(Z < z) = 0.80$): $z \approx 0.84 \implies x = 62 + (0.84)(5) = 62 + 4.2 = 66.2 \text{ cm}$.

4. Resting Heart Rate

A clinical study shows that the resting heart rate of healthy adults is normally distributed with mean $\mu = 68 \text{ bpm}$ and standard deviation of $\sigma = 8 \text{ bpm}$. What range of heart rates contains the 10% of values closest to the mean?

Solution:

Let X be the resting heart rate, $X \sim N(68, 8^2)$.

The 10% of values closest to the mean corresponds to the middle 10% of the distribution (5% on either side of the mean, leaving 45% in each tail).

This means we are looking for the boundaries at the 45th and 55th percentiles (0.50 ± 0.05).

For $P(Z < z) = 0.55$, $z \approx 0.126$ (or roughly 0.13).

For $P(Z < z) = 0.45$, $z \approx -0.126$ (or roughly -0.13).

Lower bound: $x_1 = 68 + (-0.126)(8) = 68 - 1.008 \approx 67.0 \text{ bpm}$.

Upper bound: $x_2 = 68 + (0.126)(8) = 68 + 1.008 \approx 69.0 \text{ bpm}$.

The range containing the 10% of values closest to the mean is approximately 67.0 to 69.0 bpm.