SN1 Practice Test 3

General Information and Recommendations

- Test 3 is scheduled to take place on **Thursday, December 11**.
- It covers lectures:
  - L9. The Binomial Distribution
  - L10. The Normal Distribution
  - L11. The Normal Approximation to the Binomial
  - L12. Sampling Distributions and the Central Limit Theorem
  - L13. Confidence Interval on the Mean; Single Population, Variance Known
  - L14. Confidence Interval on the Mean; Single Population, Variance Unknown
  - L15. Confidence Interval on the Population Proportion
  - L16. Hypothesis Tests on the Mean; Single Population, Variance Known
  - L17. Hypothesis Tests on the Mean; Single Population, Variance Unknown
  - L18. Hypothesis Tests on the Population Proportion
- It is strongly advised that you go **over all of the problems covered in class**, the in class exercises, take home assignments, and the questions on the practice test.
- Solutions to this practice test will be posted by Wednesday, December 3 (9:00 pm).

201-SN1-05 Probability and Statistics

Practice Test 3 - B

Winter 2025

Name:

This test consists of 10 questions.
You will have **2 hours** to complete this test.

Instructions:

- Write your answers directly on the questionnaire.
- Show all work. Your solutions will be scored on the correctness and completeness of your methods and use of proper notation as well as your answers. A final answer with no work, calculations, and/or explanations will result in a grade of zero for that questions - even if it is correct.
- Notation counts. Poor notation = Loss of marks.
- All cell phones and listening devices must be turned off. All unauthorized materials must be put away.
- Only non-graphing, non-programmable calculators are permitted.
- Give exact answers and reduce all fractions. $\sqrt{2}$ is exact, 1.41 is an approximation of $\sqrt{2}$. If using decimals, please give answers to four significant decimal places.

Note:

- Some questions will take more time, some less. Manage your time.
- Start by reading over the entire test.
- Start with a question you find easy.

Good Luck!

Marks	
1	/
2	/
3	/
4	/
5	/
6	/
7	/
8	/
9	/
10	/
Total: / (%)	

Cheating and plagiarism are serious academic offences. Anyone caught cheating, or aiding in the act of cheating, will immediately be given a mark of zero for this test, and a note will be placed in his or her file.

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1. **Master Cleanse**

A popular weight loss method among celebrities these days is the "Master Cleanse". The detox regime involves avoiding food for about 10 days, drinking a lemonade mixture flavoured with maple syrup and cayenne pepper, and consuming a gallon of saltwater on top of that. The only break from the monotony of this liquid diet is a delicious laxative before bed. Gwenyth Paltrow and Beyoncé swear by it.

"Master Cleanse" claims that you can lose an average of 10 pounds in 10 days on this diet. However, in a random sample of 14 dieters on this regime, it was found that their average weight loss in 10 days was only 8.5 pounds, with a sample standard deviation 1.75 pounds. Assume that the dieters were selected from a normally distributed population.

(a) At the 1% level of significance, does the data indicate that the amount of weight lost on Master Cleanse is less than 10 pounds in 10 days? $H_0 : \mu = 10$ lbs vs $H_a : \mu < 10$ lbs.

Given: $n = 14, \bar{x} = 8.5, s = 1.75, \alpha = 0.01$. Unknown variance $\implies$ t-test.

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{8.5 - 10}{1.75/\sqrt{14}} = \frac{-1.5}{0.4677} = -3.207$$

Degrees of freedom $df = 13$. Critical value $t_{0.01,13} = -2.650$.
Since $-3.207 < -2.650$, we **reject** H_0. There is sufficient evidence to conclude the average weight loss is less than 10 pounds.

(b) Construct appropriate interval estimate for the actual average amount of weight lost in 10 days on this diet, and explain how it can be used to support the conclusion in (a).

Since H_a is "less than", we construct a 99% Upper Confidence Bound.

$$UB = \bar{x} + t_{\alpha,n-1}\frac{s}{\sqrt{n}} = 8.5 + 2.650(0.4677) = 8.5 + 1.24 = 9.74 \text{ lbs}$$

The interval is $(-\infty, 9.74]$. Since 10 is not in this interval (10 ¿ 9.74), we reject the null hypothesis, supporting the conclusion in (a).

2. **Sleepy Lions**

Lions don't sleep in the jungle, the mighty jungle at all. The word "jungle" means dense, tropical rainforest and although it only covers 6% of the Earth's land, nearly 57% of its species live there. Lions are not one of them - almost all lions live on the savannah of sub-Saharan Africa. Not only do lions not live in the jungle, "tonight" is the least likely time for them to be sleeping. Outside the mating season, they spend about 20 hours sleeping, two hours walking and just under an hour eating.

According to zoological records, the average lifespan of a male lion is 11.25 years, with a known population standard deviation of 2.89 years. However, some conservationists believe that lions in a particular region of South Sudan may live longer than average.

To investigate this, researchers took a random sample of 35 male lions from the area and found their mean lifespan to be 12.5 years.

(a) At the 2.5% level of significance, does the data indicate that male lions in this region tend to live longer than 11.25 years? $H_0 : \mu = 11.25$ vs $H_a : \mu > 11.25$.
Given: $\mu_0 = 11.25, \sigma = 2.89, n = 35, \bar{x} = 12.5$. Known variance $\implies$ Z-test.

$$Z = \frac{12.5 - 11.25}{2.89/\sqrt{35}} = \frac{1.25}{0.4885} = 2.56$$

Critical value $Z_{0.025} = 1.96$.
Since $2.56 > 1.96$, we **reject** H_0. The data indicates they live longer.

(b) Construct a one-sided interval estimate and explain how it can be used to support the conclusion obtained in (a). Construct a 97.5% Lower Confidence Bound ($Z_{0.025} = 1.96$).

$$LB = \bar{x} - Z\frac{\sigma}{\sqrt{n}} = 12.5 - 1.96(0.4885) = 12.5 - 0.96 = 11.54 \text{ years}$$

We are 97.5% confident the mean is greater than 11.54. Since 11.25 is not in the interval $[11.54, \infty)$, we reject H_0.

(c) What is the Type II error in the context of the problem? A Type II error would occur if we conclude that the lions in this region **do not** live longer than average (fail to reject H_0), when in reality they **do** live longer (H_a is true).

3. **Liquid Soap**

A machine is set to fill 16-ounce bottle with liquid soap. When the machine has been properly calibrated, the average amount of soap dispensed into each bottle is 16 ounces, with a known standard deviation of 0.25 ounces. A quality control inspector selects 55 bottles from a production run and finds that the average volume of soap dispensed in the containers to be 15.93 ounces.

(a) At the 1% level of significance, does the data indicate that the average amount of soap dispensed into the containers by the machine is different from 16 ounces? What is the actual probability of committing a Type I error? $H_0 : \mu = 16$ vs $H_a : \mu \neq 16$. $\sigma = 0.25, n = 55, \bar{x} = 15.93$. Z-test.

$$Z = \frac{15.93 - 16}{0.25/\sqrt{55}} = \frac{-0.07}{0.0337} = -2.08$$

Critical values for $\alpha = 0.01$ (two-tailed) are ± 2.576.
Since $|-2.08| < 2.576$, we **fail to reject** H_0.
The probability of committing a Type I error is fixed at $\alpha = \mathbf{0.01}$.

(b) Construct and explain how an appropriate confidence interval/bound could be used to support the conclusion of the test obtained in part (a). Construct a 99% two-sided Confidence Interval.

$$\bar{x} \pm 2.576(0.0337) = 15.93 \pm 0.087$$

$$[15.84, 16.02]$$

Since 16 is within the interval, we cannot reject the null hypothesis.

(c) Based on your results from (a) and (b), does the machine require recalibration? Explain in one or two sentences. No, the machine does not require recalibration. The data does not provide sufficient evidence that the mean fill volume is different from the target of 16 ounces.

4. **Dinosaurs**

In the 1940s, some scientists believed Venus was home to dinosaurs because it was at an earlier stage of evolution than Earth.

A science historian claims that more than 60% of science fiction articles from the 1940s mentioned prehistoric creatures when describing life on Venus. To investigate this claim, a researcher randomly selects 200 articles from that decade and finds that 135 of them mention prehistoric creatures.

(a) At the 5% level of significance, test the historian's claim. What is the $P-value$ of this test? $H_0 : p = 0.60$ vs $H_a : p > 0.60$.
$\hat{p} = \frac{135}{200} = 0.675$.

$$Z = \frac{0.675 - 0.60}{\sqrt{\frac{0.6(0.4)}{200}}} = \frac{0.075}{\sqrt{0.0012}} = \frac{0.075}{0.0346} = 2.17$$

P-value: $P(Z > 2.17) = 1 - 0.9850 = \mathbf{0.0150}$.
Since $0.015 < 0.05$, we **reject** H_0. The historian's claim is supported.

(b) Construct and explain how an appropriate confidence interval/bound could be used to support the conclusion of the test obtained in part (a). Construct a 95% Lower Confidence Bound ($Z_{0.05} = 1.645$).

$$LB = \hat{p} - Z\sqrt{\frac{\hat{p}\hat{q}}{n}} = 0.675 - 1.645\sqrt{\frac{0.675(0.325)}{200}}$$

$$LB = 0.675 - 1.645(0.033) = 0.675 - 0.054 = 0.621$$

Interval: $[0.621, 1]$. Since 0.60 is not in the interval, we reject H_0.

(c) What is the minimum sample size required to estimate the true proportion of science fiction articles mentioning prehistoric creatures, if we want to test the claim at the 5% level of significance but do not have a preliminary estimate for p, and we want the margin of error to be no more than 15%? Since there is no preliminary estimate, use $p = 0.5$. $E = 0.15$. Assuming 95% confidence ($Z = 1.96$):

$$n = 0.25\left(\frac{Z}{E}\right)^2 = 0.25\left(\frac{1.96}{0.15}\right)^2 = 0.25(170.7) = 42.67$$

Round up to **43**.

5. **Charles Darwin**

When Charles Darwin published a diary of his baby son's milestones, he started a trend among naturalists who were also fathers to take more of an interest in their children's development.

Records indicate that the birth weights of Victorian-era babies were approximately normally distributed with a mean of 3.4 kg and a standard deviation of 0.5 kg

(a) What is the probability that a randomly selected baby from the Victorian-era weighs between 2.8 kg and 3.8 kg?

$$P(2.8 < X < 3.8) = P\left(\frac{2.8 - 3.4}{0.5} < Z < \frac{3.8 - 3.4}{0.5}\right)$$

$$= P(-1.2 < Z < 0.8) = 0.7881 - 0.1151 = \mathbf{0.6730}$$

(b) What is the probability that out of 6 randomly selected Victorian-era babies, at least 2 of them weigh less than 4.54 kg? First, find $p = P(X < 4.54)$. $Z = (4.54 - 3.4)/0.5 = 2.28$. $p = P(Z < 2.28) = 0.9887$. Using Binomial $n = 6, p = 0.9887$: $P(X \geq 2) = 1 - P(0) - P(1)$. Given p is almost 1, the probability of at least 2 is **approximately 1**. (Exact calculation yields > 0.9999).

(c) If 36 Victorian-era babies are selected, what is the probability that their average weight is below 3.25 kg? Sampling distribution: $n = 36$. $\sigma_{\bar{x}} = 0.5/\sqrt{36} = 0.0833$.

$$P(\bar{X} < 3.25) = P\left(Z < \frac{3.25 - 3.4}{0.0833}\right) = P(Z < -1.80) = \mathbf{0.0359}$$

(d) If 49 Victorian-era babies are selected, what is the probability that their average weight is more than 3.575 kg? Sampling distribution: $n = 49$. $\sigma_{\bar{x}} = 0.5/\sqrt{49} = 0.0714$.

$$P(\bar{X} > 3.575) = P\left(Z > \frac{3.575 - 3.4}{0.0714}\right) = P(Z > 2.45) = 1 - 0.9929 = \mathbf{0.0071}$$

6. **ADHD**

A recent study claims that American doctors are 14% more likely to diagnose children with ADHD on Halloween, raising concerns about the influence of timing and social expectations on medical decisions.

The historical proportion of ADHD diagnoses in children is approximately 10%. On Halloween, a random sample of 300 medical records shows that 39 children were diagnosed with ADHD.

(a) At the 1% level of significance, test the claim that the proportion of diagnoses is different from 10% on Halloween. $H_0 : p = 0.10$ vs $H_a : p \neq 0.10$.
$\hat{p} = 39/300 = 0.13$.

$$Z = \frac{0.13 - 0.10}{\sqrt{\frac{0.1(0.9)}{300}}} = \frac{0.03}{\sqrt{0.0003}} = \frac{0.03}{0.01732} = 1.73$$

P-value $= 2 \times P(Z > 1.73) = 2(1 - 0.9582) = 2(0.0418) = \mathbf{0.0836}$.
Since $0.0836 > 0.01$, we **fail to reject** H_0.

(b) What is the highest level significance for which you would be able to reject the null hypothesis. We reject when $\alpha \geq$ P-value. Therefore, the highest significance level (assuming standard levels) where we fail to reject is below 0.0836. The phrasing implies finding the P-value threshold: **0.0836 (or 8.36%)**.

(c) Construct and explain how an appropriate confidence interval/bound could be used to support the conclusion of the test obtained in part (a). Construct a 99% Confidence Interval ($Z_{0.005} = 2.576$).

$$CI = 0.13 \pm 2.576(0.0173) = 0.13 \pm 0.045$$

$$[0.085, 0.175]$$

Since 0.10 falls within this interval, we cannot reject the null hypothesis.

(d) A researcher wants to estimate the true proportion of ADHD diagnoses on Halloween with a margin of error no greater than 3% at the 99% level of confidence. A preliminary study suggests the proportion may be around 13%. What is the minimum sample size the researcher should use to achieve this level of precision? $\hat{p} = 0.13, E = 0.03, Z_{0.005} = 2.576$.

$$n = \hat{p}\hat{q}\left(\frac{Z}{E}\right)^2 = 0.13(0.87)\left(\frac{2.576}{0.03}\right)^2$$

$$n = 0.1131(7373.9) = 833.99$$

Round up to **834**.

7. **Confiscated Crops**

In the past three years, Essex Police have been growing high-quality weed. When they seize undeveloped cannabis plants, they often grow them to maturity — otherwise, the plants may be considered worthless and not justify the confiscation of a drug dealer's assets. An internal review claims that the average yield per mature plant is 52 grams. A random sample of 10 plants grown to maturity produced the following yields (in grams):

47	55	60	49	53	57	51	58	48	54

(a) Calculate the mean and the standard deviation of the sample data. Sum = 532. $\bar{x} = 532/10 = \mathbf{53.2}$.
Sum of squared deviations $SS = (47 - 53.2)^2 + ... = 38.44 + 3.24 + 46.24 + 17.64 + 0.04 + 14.44 + 4.84 + 23.04 + 27.04 + 0.64 = 175.6$.
$s^2 = 175.6/(10 - 1) = 19.51$. $s = \sqrt{19.51} = \mathbf{4.42}$.

(b) Using your results from (a), test the claim that the average yield per plant is 52 grams. Use a significance level of $\alpha = 0.05$ and estimate the P-value. $H_0 : \mu = 52$ vs $H_a : \mu \neq 52$. (Two-tailed).

$$t = \frac{53.2 - 52}{4.42/\sqrt{10}} = \frac{1.2}{1.398} = 0.86$$

$df = 9$. Looking at T-table row 9: $t = 0.86$ is less than $t_{0.10} = 1.383$. So one-tail $P > 0.10$, and two-tail $P > 0.20$. We **fail to reject** H_0.

(c) Construct and explain how a confidence interval or bound could support the conclusion from part (b). 95% Confidence Interval ($t_{0.025,9} = 2.262$).

$$53.2 \pm 2.262(1.398) = 53.2 \pm 3.16$$

$$[50.04, 56.36]$$

Since 52 is inside the interval, we do not reject the null hypothesis.

(d) Clearly state what a Type I and Type II error would mean in the context of this problem. **Type I:** Concluding the average yield is different from 52g when it is actually 52g.
Type II: Concluding the average yield is 52g when it is actually different.

8. For each question below, select **all** the statements that are **correct**. Each question has **at least one correct answer, but not necessarily all options are correct**. You will receive **full credit** if and only if you select all correct answers and **no incorrect answers**. Selecting an incorrect option or missing a correct option may result in **partial credit or no credit**.

   (a) Suppose a medical test is used to determine whether a person has a certain disease. The null hypothesis H_0, is that the person does not have the disease. Which of the following statements are correct?

   X ☐ A Type I error occurs if the test concludes the person **has** the disease when they actually do **not**.

   X ☐ A Type II error occurs if the test concludes the person does **not** have the disease when they actually **do**.

   X ☐ Reducing the significance level α decreases the probability of a Type I error.

   ☐ Reducing the significance level α always decreases the probability of a Type II error.

   ☐ It is possible to simultaneously reduce both Type I and Type II error rates to zero by choosing a very low significance level.

   (b) Suppose a 95% confidence interval for the mean weight of a certain species of bird is between 2.4 kg and 3.0 kg. Which of the following statements are correct?

   X ☐ If we constructed many such intervals from repeated samples, about 95% of them would contain the true mean.

   ☐ There is a 95% chance that the true mean is in the interval $(2.4, 3.0)$.

   X ☐ The confidence interval provides a plausible range for the population mean.

   ☐ A larger sample size would result in a wider confidence interval.

   X ☐ We are 95% confident that the true mean weight of the species lies between 2.4 kg and 3.0 kg.

   (c) Suppose the distribution of weights for a species of fish is right-skewed with a mean of 4.5 kg and a standard deviation of 1.2 kg. A biologist takes random samples of size $n = 40$ and calculates the sample mean weight $\bar{x}$ for each sample. Which of the following statements are correct?

   X ☐ The standard deviation of the sampling distribution of $\bar{x}$ is $\frac{1.2}{\sqrt{40}}$.

   ☐ The sampling distribution will be skewed right, just like the population distribution.

   ☐ Increasing the sample size would increase the variability of the sample means.

   X ☐ The distribution of the sample means $\bar{x}$ will be approximately normal.

   X ☐ The mean of the sampling distribution of $\bar{x}$ will be approximately 4.5 kg.

   (d) A recent study reports that 60% of all adults in a city recycle regularly. A researcher believes the true proportion has increased. They collect a random sample and conduct a hypothesis test with $H_0 : p = 0.60$ and $H_a : p > 0.60$. The test yields a p-value of 0.03. Assume a significance level of $\alpha = 0.05$. Which of the following statements are correct?

   X ☐ The null hypothesis should be rejected.

   X ☐ There is statistically significant evidence that the true proportion of adults who recycle is greater than 60%.

   ☐ The result is not significant at the 5% level, so we fail to reject the null hypothesis.

   ☐ The p-value of 0.03 means there's a 3% chance the null hypothesis is true.

   X ☐ If the true proportion were actually 60%, there would be a 3% chance of getting a sample result this extreme or more by random chance.

9. **A Beetle Called Hitler**

Anophthalmus hitleri is a blind beetle found only in five caves in Slovenia. Named after Hitler in 1933, it is now endangered not because of pollution or pressure from predators, but from collectors of Nazi memorabilia3. Suppose that the lifespan of these beetles are normally distributed with a mean of 90 days and a standard deviation of 15 days.

(a) What is the probability that a randomly selected Anophthalmus hitleri will live longer than 140 days?

$$P(X > 140) = P\left(Z > \frac{140-90}{15}\right) = P(Z > 3.33) = 1 - 0.9996 = \mathbf{0.0004}$$

(b) What is the probability that a randomly selected Anophthalmus hitleri will survive longer than 70 days but not more than 110 days?

$$P(70 < X < 110) = P\left(\frac{70-90}{15} < Z < \frac{110-90}{15}\right)$$

$$= P(-1.33 < Z < 1.33) = 0.9082 - 0.0918 = \mathbf{0.8164}$$

(c) The bottom 5% of beetles will survive for how many days. Find x such that $P(X < x) = 0.05$. $Z = -1.645$.

$$x = 90 + (-1.645)(15) = 90 - 24.675 = \mathbf{65.325} \text{ days}$$

(d) Out of 22 beetles, what is the probability that at least 19 of them will survive more than 100 days? First find $p = P(X > 100)$. $p = P(Z > \frac{100-90}{15}) = P(Z > 0.67) = 1 - 0.7486 = 0.2514$. Binomial: $n = 22, p = 0.2514$. Expected value ≈ 5.5. $P(Y \geq 19)$ is extremely small (approaching zero). Calculated: $\sum_{k=19}^{22} \binom{22}{k} (0.2514)^k (0.7486)^{22-k} \approx$ $\mathbf{0.0000}$

(e) If a beetle lives at least 70 days, what is the probability that it lives between 80 and 100 days? Conditional Probability $P(80 < X < 100 | X > 70) = \frac{P(80<X<100 \cap X>70)}{P(X>70)}$. Since $(80, 100)$ is inside $(70, \infty)$, numerator is $P(80 < X < 100)$. $P(80 < X < 100) = P(-0.67 < Z < 0.67) = 0.7486 - 0.2514 = 0.4972$. $P(X > 70) = P(Z > -1.33) = 0.9082$.

$$\frac{0.4972}{0.9082} = \mathbf{0.5475}$$

10. **Traffic Tickets**

When traffic cameras were first installed in California, a man received a speeding ticket in the mail, complete with a photo as evidence. Upset, he mailed back a photo of the fine instead of the actual money. Weeks later, he received another photo — this time of handcuffs. Suppose that 15% of drivers who speed past a traffic camera receive a ticket in the mail.

(a) What is the probability that out of twelve drivers who sped past a camera, three or four will receive a ticket in the mail? Binomial $n = 12, p = 0.15$.

$$P(3) = \binom{12}{3}(0.15)^3(0.85)^9 = 220(0.003375)(0.2316) = 0.1720$$

$$P(4) = \binom{12}{4}(0.15)^4(0.85)^8 = 495(0.000506)(0.2725) = 0.0683$$

Sum: $0.1720 + 0.0683 = \mathbf{0.2403}$

(b) What is the probability that out of fifteen drivers who sped past a camera, at least two will receive a ticket in the mail? Binomial $n = 15, p = 0.15$. $P(\geq 2) = 1 - P(0) - P(1)$.

$$P(0) = (0.85)^{15} = 0.0874$$

$$P(1) = 15(0.15)(0.85)^{14} = 0.2312$$

$$1 - (0.0874 + 0.2312) = 1 - 0.3186 = \mathbf{0.6814}$$

(c) What is the probability that out of 1000 drivers who sped past a camera, at least 870 will not receive a ticket in the mail? Let Y be drivers who do NOT receive a ticket. $p = 0.85, n = 1000$. Mean $\mu = 850$. $\sigma = \sqrt{1000(0.85)(0.15)} = \sqrt{127.5} = 11.29$. Normal approx with continuity correction $(Y \geq 869.5)$.

$$Z = \frac{869.5 - 850}{11.29} = 1.73$$

$P(Z > 1.73) = 1 - 0.9582 = \mathbf{0.0418}$

(d) What is the probability that out of 1000 drivers who sped past a camera, at between 820 and 860 (exclusive) will not receive a ticket in the mail? We want $P(820 < Y < 860)$. With continuity correction: $P(820.5 \leq Y \leq 859.5)$.

$$Z_1 = \frac{820.5 - 850}{11.29} = -2.61$$

$$Z_2 = \frac{859.5 - 850}{11.29} = 0.84$$

$P(-2.61 < Z < 0.84) = 0.7995 - 0.0045 = \mathbf{0.7950}$