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Date: 201-316-VA

In Class Exercise # 10: Rules of probability

1. **Say What?**

   In 1993, the Barbie Liberation Organization swapped the voice boxes out of 400 talking G.I. Joe dolls, with those of talking Barbies, and then returned them to stores. Unsuspecting kids who bought the toys heard Barbie say "Vengeance is mine!" and G.I. Joe say, "I love shopping!"[1]

   At a toy store, there are 20 Barbie and 30 G.I. Joe dolls on the shelves. Fifteen of the Barbies and 18 of the Joe dolls have had their voice boxes switched out.

   (a) If one G.I Joe doll and one Barbie doll are selected at random, what is the probability both had their voice boxes switched out?

   Let J_S be the event a G.I. Joe has a switched voice box. Let B_S be the event a Barbie has a switched voice box.

   $$P(J_S) = \frac{18}{30} = \frac{3}{5} \qquad P(B_S) = \frac{15}{20} = \frac{3}{4}$$

   These are independent events.

   $$P(J_S \cap B_S) = P(J_S) \times P(B_S) = \frac{3}{5} \times \frac{3}{4} = \frac{9}{20} = 0.45$$

   (b) If one G.I Joe doll and one Barbie doll are selected at random, what is the probability neither had their voice boxes switched out?

   Let J_N be the event a G.I. Joe is not switched ($J_N = J'_S$). Let B_N be the event a Barbie is not switched ($B_N = B'_S$).

   $$P(J_N) = 1 - \frac{18}{30} = \frac{12}{30} = \frac{2}{5} \qquad P(B_N) = 1 - \frac{15}{20} = \frac{5}{20} = \frac{1}{4}$$

   $$P(J_N \cap B_N) = P(J_N) \times P(B_N) = \frac{2}{5} \times \frac{1}{4} = \frac{2}{20} = \frac{1}{10} = 0.1$$

   (c) If one G.I Joe doll and one Barbie doll are selected at random, what is the probability at least one of them had their voice boxes switched out?

   This is the complement of the event in part (b) (neither switched).

   $$P(\text{At least one switched}) = 1 - P(\text{Neither switched})$$

   $$P(\text{At least one switched}) = 1 - P(J_N \cap B_N) = 1 - 0.1 = 0.9$$

   (d) If three Joe dolls were purchased by three different clients, what is the probability all three have had their voice boxes switched out?

   This is sampling without replacement.

   $$P(\text{All 3 switched}) = P(\text{1st } J_S) \times P(\text{2nd } J_S|\text{1st } J_S) \times P(\text{3rd } J_S|\text{1st, 2nd } J_S)$$

   $$= \frac{18}{30} \times \frac{17}{29} \times \frac{16}{28} = \frac{4896}{24360} = \frac{204}{1015} \approx 0.201$$

2. **1914 Christmas Truce**

   In the early months of World War I, an unofficial ceasefire was declared along the Western Front. In the weeks leading up to Christmas, German and British soldiers took the opportunity to exchange gifts and play soccer with each other. On the German side, troops put up a sign which read "Gott mit uns" ("God with us"). In response, the British put up a cheeky sign which said "We've got mittens, too"[2].

   At a local store, 85% of the mittens are made in Germany while the rest are made in England. Suppose that 4% of the British mittens have an imperfection in them, while the same could be said about 6% of the German ones. What is the probability that a randomly selected pair of mittens

   (a) has an imperfection?

   Let G = German, E = English, I = Imperfection, I' = No Imperfection. Given: $P(G) = 0.85$, $P(E) = 0.15$ Given: $P(I|E) = 0.04$, $P(I|G) = 0.06$

   Using the Law of Total Probability:

   $$P(I) = P(I|G)P(G) + P(I|E)P(E) = (0.06)(0.85) + (0.04)(0.15) = 0.057$$

[1] https://en.wikipedia.org/wiki/Barbie_Liberation_Organization
[2] http://www.ppu.org.uk/remembrance/xmas.html

(b) is free of imperfections and German made?

We want $P(I' \cap G)$. First, find $P(I'|G) = 1 - P(I|G) = 1 - 0.06 = 0.94$.

$$P(I' \cap G) = P(I'|G)P(G) = (0.94)(0.85) = 0.799$$

(c) was British made if it had an imperfection?

We want $P(E|I)$ (Bayes' Theorem).

$$P(E|I) = \frac{P(E \cap I)}{P(I)} = \frac{P(I|E)P(E)}{P(I)}$$

From part (a), $P(I) = 0.057$. $P(E \cap I) = P(I|E)P(E) = (0.04)(0.15) = 0.006$.

$$P(E|I) = \frac{0.006}{0.057} = \frac{6}{57} = \frac{2}{19} \approx 0.1053$$

(d) was German made if it had no imperfections?

We want $P(G|I')$.

$$P(G|I') = \frac{P(G \cap I')}{P(I')}$$

From part (b), $P(G \cap I') = 0.799$. $P(I') = 1 - P(I) = 1 - 0.057 = 0.943$.

$$P(G|I') = \frac{0.799}{0.943} \approx 0.8473$$

3. **The Shaggy Defense**

The Shaggy defense is a legal strategy in which the defendant flatly denies guilt even though there is overwhelming evidence against them - particularly in the form of a video recording[3]. The hallmarks of the defense involves a refusal to engage with the evidence and strenuous denial that it was them who committed the act. The term is derived from Shaggy's 2000 single "It Wasn't Me" in which the song describes a man asking a friend what to do after his girlfriend catches him cheating on her. The friend's advice was to simply deny that it was him.

Lawyer, Saul Goodman, uses the Shaggy defense in 30% of the cases that he is assigned to. The rest of the time, he uses the Chewbacca defense[4]. Whenever he uses the Shaggy defense, it results in a not guilty verdict 40% of the time, and a guilty verdict the rest of the time. If he were to use the Chewbacca defense, then the probability of getting a not guilty verdict for his client is 80% and a guilty verdict the rest of the time.

(a) What proportion of Saul's cases end in a not guilty verdict?

Let S = Shaggy defense, C = Chewbacca defense, NG = Not Guilty, G = Guilty. Given: $P(S) = 0.30$, $P(C) = 0.70$ Given: $P(NG|S) = 0.40$, $P(G|S) = 0.60$ Given: $P(NG|C) = 0.80$, $P(G|C) = 0.20$

Using the Law of Total Probability:

$$P(NG) = P(NG|S)P(S) + P(NG|C)P(C)$$

$$P(NG) = (0.40)(0.30) + (0.80)(0.70) = 0.12 + 0.56 = 0.68$$

(b) If a client was found guilty, what is the probability that he used the Shaggy defense?

We want $P(S|G)$ (Bayes' Theorem).

$$P(S|G) = \frac{P(S \cap G)}{P(G)} = \frac{P(G|S)P(S)}{P(G)}$$

First, find $P(G) = 1 - P(NG) = 1 - 0.68 = 0.32$.
$P(S \cap G) = P(G|S)P(S) = (0.60)(0.30) = 0.18$.

$$P(S|G) = \frac{0.18}{0.32} = \frac{18}{32} = \frac{9}{16} = 0.5625$$

(c) If a client was found to be not guilty, what is the probability that the Chewbacca defense was used?

We want $P(C|NG)$.

$$P(C|NG) = \frac{P(C \cap NG)}{P(NG)} = \frac{P(NG|C)P(C)}{P(NG)}$$

From part (a), $P(NG) = 0.68$. $P(C \cap NG) = P(NG|C)P(C) = (0.80)(0.70) = 0.56$.

$$P(C|NG) = \frac{0.56}{0.68} = \frac{56}{68} = \frac{14}{17} \approx 0.8235$$

(d) If a case from Saul were to be randomly selected, what is the probability that he used the Shaggy defense or got a not guilty verdict for his client?

We want $P(S \cup NG)$. Using the General Addition Rule:

$$P(S \cup NG) = P(S) + P(NG) - P(S \cap NG)$$

$P(S) = 0.30$ $P(NG) = 0.68$ (from part a) $P(S \cap NG) = P(NG|S)P(S) = (0.40)(0.30) = 0.12$

$$P(S \cup NG) = 0.30 + 0.68 - 0.12 = 0.86$$

[3]https://en.wikipedia.org/wiki/Shaggy_defense
[4]See page 69