Name: ______________________________ Business Data Analysis
Date: 201-316-VA

In Class Exercise # 11: The Gaussian Distribution

1. **Basic Calculation** Suppose that X is a normally distributed random variable with $\mu = 50$ and $\sigma = 5$. Determine the following values:

   (a) $P(X \leq 65) : Z = \frac{65-50}{5} = 3.00 \implies P(Z \leq 3.00) = 0.9987$

   (b) $P(X < 40) : Z = \frac{40-50}{5} = -2.00 \implies P(Z \leq -2.00) = 0.0228$

   (c) $P(X \geq 58) : Z = \frac{58-50}{5} = 1.60 \implies P(Z \geq 1.60) = 1 - P(Z \leq 1.60) = 1 - 0.9452 = 0.0548$

   (d) $P(X > 67) : Z = \frac{67-50}{5} = 3.40 \implies P(Z \geq 3.40) = 1 - P(Z \leq 3.40) = 1 - 0.9997 = 0.0003$

   (e) $P(40 < X \leq 60) : Z_1 = \frac{40-50}{5} = -2.00, Z_2 = \frac{60-50}{5} = 2.00 \implies P(-2.00 \leq Z \leq 2.00) = P(Z \leq 2.00) - P(Z \leq -2.00) = 0.9772 - 0.0228 = 0.9544$

   (f) $P(40 < X < 45) : Z_1 = \frac{40-50}{5} = -2.00, Z_2 = \frac{45-50}{5} = -1.00 \implies P(-2.00 \leq Z \leq -1.00) = P(Z \leq -1.00) - P(Z \leq -2.00) = 0.1587 - 0.0228 = 0.1359$

2. **Is it Art?** In Scotland, a group of students went to a modern art gallery and left a pineapple in an empty exhibit to see if people would think that it was art. When they returned four days later, not only was the pineapple still there, but it had also been covered with a glass case. The Cayenne Pineapple is the most popular variety of pineapple sold in Scotland. The height of a mature Cayenne pineapple is normally distributed with a mean of 30 cm and a standard deviation of 4.5 cm.

   (a) What is the probability that a randomly selected pineapple will have a height of at least 35 centimetres?
   $Z = \frac{35-30}{4.5} \approx 1.11 \implies P(Z \geq 1.11) = 1 - P(Z \leq 1.11) = 1 - 0.8665 = 0.1335$

   (b) What is the probability that a randomly selected Cayenne pineapple will have a height of less than 20.5 centimetres?
   $Z = \frac{20.5-30}{4.5} \approx -2.11 \implies P(Z \leq -2.11) = 0.0174$

   (c) What is the probability that a randomly selected pineapple will have a height that is between 19 and 37 centimetres?
   $Z_1 = \frac{19-30}{4.5} \approx -2.44, Z_2 = \frac{37-30}{4.5} \approx 1.56 \implies P(-2.44 \leq Z \leq 1.56) = P(Z \leq 1.56) - P(Z \leq -2.44) = 0.9406 - 0.0073 = 0.9333$

   (d) 14% of all Cayenne pineapples are taller than how many centimetres?
   $P(X > x) = 0.14 \implies P(Z > z) = 0.14 \implies P(Z \leq z) = 0.86$. From table, $Z \approx 1.08$.
   $X = \mu + Z\sigma = 30 + (1.08)(4.5) = 30 + 4.86 = 34.86$ cm

3. **Where's my food?** In 2015, a group of hackers took down a Papa John's website because their pizza was 2 hours late.

   (a) If the amount of time for Papa John's to deliver a pizza is normally distributed with an average 30 minutes with a standard deviation of 5 minutes, what is the probability that

      i. a pizza will be delivered in less than 40 minutes?
      $Z = \frac{40-30}{5} = 2.00 \implies P(Z \leq 2.00) = 0.9772$

      ii. it will take at least 45 minutes for a pizza to be delivered?
      $Z = \frac{45-30}{5} = 3.00 \implies P(Z \geq 3.00) = 1 - P(Z \leq 3.00) = 1 - 0.9987 = 0.0013$

      iii. it will take between 20 and 40 minutes for the pizza to be delivered?
      $Z_1 = \frac{20-30}{5} = -2.00, Z_2 = \frac{40-30}{5} = 2.00 \implies P(-2.00 \leq Z \leq 2.00) = P(Z \leq 2.00) - P(Z \leq -2.00) = 0.9772 - 0.0228 = 0.9544$

   (b) In how many minutes will 90% of all pizzas be delivered by?
   $P(X \leq x) = 0.90 \implies P(Z \leq z) = 0.90$. From table, $Z \approx 1.28$.
   $X = \mu + Z\sigma = 30 + (1.28)(5) = 30 + 6.4 = 36.4$ minutes

   (c) Only 1% of pizzas take longer than how many minutes to be delivered?
   $P(X > x) = 0.01 \implies P(Z \leq z) = 0.99$. From table, $Z \approx 2.33$.
   $X = \mu + Z\sigma = 30 + (2.33)(5) = 30 + 11.65 = 41.65$ minutes

   (d) Suppose that three people call Papa John's for a pizza. Assuming that each call is independent, and that each car is assigned a single order, what is the probability that

      i. all three orders are delivered in less than 35 minutes?
      Let $p = P(X < 35)$. $Z = \frac{35-30}{5} = 1.00 \implies p = P(Z < 1.00) = 0.8413$.
      Let Y be the number of orders < 35 min. $Y \sim Bin(n = 3, p = 0.8413)$.
      $P(Y = 3) = (0.8413)^3 \approx 0.5955$

      ii. only one client receives their pizza in under 35 minutes?
      Using $p = 0.8413$ and $q = 1 - p = 0.1587$.
      $P(Y = 1) = \binom{3}{1} p^1 q^2 = 3(0.8413)(0.1587)^2 \approx 0.0636$

4. **Mercedes-Benz** In 2010, a Colorado money manager maimed a cyclist in a hit-and-run. He escaped felony charges by arguing that the smell of his new Mercedes-Benz had aggravated his sleep apnea - causing him to be knocked out temporarily.[8]

(a) Engines for Benz's AMG line are assembled at a factory in Stuttgart, Germany. The assembly times for CLK-55 engines follows a normal distribution with a mean of 55 minutes and a standard deviation of 4 minutes. The factory closes at 5 p.m. every day. If a worker begins assembling an engine at 4 p.m., what is the probability that he or she will finish the job before the factory closes for the day?
Time available = 60 minutes. We need $P(X \leq 60)$.
$Z = \frac{60-55}{4} = 1.25 \implies P(Z \leq 1.25) = 0.8944$

(b) The Silver Star dealer in Montreal provides oil and lube service for Mercedes. It is known that the mean time taken for oil and lube service is 15 minutes per car and the standard deviation is 2.4 minutes. The management wants to promote the business by guaranteeing a maximum waiting time for its customers. If a customers car is not serviced within that period, the customer will get deep discount for the service. The company wants to limit this discount to at most 5% of its customers. What should the maximum guaranteed waiting time be?
We need to find x such that $P(X > x) = 0.05$, or $P(X \leq x) = 0.95$.
$P(Z \leq z) = 0.95 \implies Z \approx 1.645$.
$X = \mu + Z\sigma = 15 + (1.645)(2.4) = 15 + 3.948 \approx 18.95$ minutes.
The guaranteed time should be 18.95 minutes.