Name: ______________________________ Business Data Analysis
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In Class Exercise # 12: Sampling Distribution

1. **There's A Saint for Everything**

   St Apollonia is the patron saint of dentists and St Réne Goupil is the patron saint of anaesthesia. St Adrian is the patron saint of arms dealers, while St Godelieve is the patron saint of abusive spouses, difficult marriages, in-law problems, healthy throats, and paradoxically, throat diseases. And there's St. Nicholas (Santa Claus) who according to legend, resurrected three little boys who were cut up and pickled in a barrel to be served as bacon. He is the patron saint of children, sailors, murderers, thieves, and in somewhat poor taste, barrel makers as well. During the Christmas holiday, it is known that the average amount of money spent on gifts for small children is normally distributed with a mean of \$200 with a standard deviation of \$14.

   (a) What is the probability that a randomly selected individual will have spent more than \$185 on Christmas gifts for small children? We are given $X \sim N(\mu = 200, \sigma = 14)$. We want to find $P(X > 185)$.

   $$Z = \frac{X - \mu}{\sigma} = \frac{185 - 200}{14} = \frac{-15}{14} \approx -1.07$$

   $$P(X > 185) = P(Z > -1.07) = 1 - P(Z < -1.07) = 1 - 0.1423 = 0.8577$$

   The probability is 85.77%.

   (b) Suppose that your are standing in line with your purchases, and there are four people in front of you waiting to pay. What is the probability that mean amount spent on gifts is greater \$185 for this group of people? Here we have a sample size $n = 4$. We are looking for the probability of the sample mean, $P(\bar{X} > 185)$. The sampling distribution is $\bar{X} \sim N(\mu_{\bar{X}}, \sigma_{\bar{X}})$. The mean of the sampling distribution is $\mu_{\bar{X}} = \mu = 200$. The standard error (standard deviation of the sampling distribution) is:

   $$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{14}{\sqrt{4}} = \frac{14}{2} = 7$$

   Now we find the Z-score for the sample mean:

   $$Z = \frac{\bar{X} - \mu_{\bar{X}}}{\sigma_{\bar{X}}} = \frac{185 - 200}{7} = \frac{-15}{7} \approx -2.14$$

   $$P(\bar{X} > 185) = P(Z > -2.14) = 1 - P(Z < -2.14) = 1 - 0.0162 = 0.9838$$

   The probability is 98.38%.

   (c) In another line, there are nine people waiting to pay for their purchases. What is the probability that the mean amount spent on gifts will exceed \$185 for this group of people? Here the sample size is $n = 9$. We are looking for $P(\bar{X} > 185)$. The mean of the sampling distribution is $\mu_{\bar{X}} = \mu = 200$. The standard error is:

   $$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{14}{\sqrt{9}} = \frac{14}{3} \approx 4.67$$

   Now we find the Z-score for the sample mean:

   $$Z = \frac{\bar{X} - \mu_{\bar{X}}}{\sigma_{\bar{X}}} = \frac{185 - 200}{14/3} = \frac{-15}{14/3} = \frac{-45}{14} \approx -3.21$$

   $$P(\bar{X} > 185) = P(Z > -3.21) = 1 - P(Z < -3.21) = 1 - 0.0007 = 0.9993$$

   The probability is 99.93%.

   (d) Compare the answers obtained in part (b) and (c). Which probability is is greater? Why would that to be the case? The probability in part (c) (0.9993) is greater than the probability in part (b) (0.9838). This is because as the sample size n increases (from 4 to 9), the standard error of the mean ($\sigma_{\bar{X}} = \sigma/\sqrt{n}$) decreases (from 7 to 4.67). A smaller standard error means the sampling distribution of $\bar{X}$ becomes narrower (less spread out) and more concentrated around the true population mean $\mu = 200$.

   Since the value of interest, \$185, is below the mean, a narrower distribution will have less of its tail area to the left of \$185 and consequently more of its probability mass to the right of \$185.