Name: ______________________________ Business Data Analysis
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In Class Exercise # 9: Rules of probability

1. **Bread and Cheese**

   In 1381, anti-German mobs in England killed anyone who couldn't say 'bread and cheese' in an English accent.

   At a wine-and-cheese event, there are 100 people in attendance. 40 have cheese on their plates, 25 have bread on their plates, and 15 have both bread and cheese on their plates. Find the probability that a person chosen at random

   (a) has cheese or bread on their plates. Let C be the event a person has cheese and B be the event a person has bread. We are given: $N = 100$ Number(C) $= 40 \implies P(C) = 40/100 = 0.40$ Number(B) $= 25 \implies P(B) = 25/100 = 0.25$ Number$(C \cap B) = 15 \implies P(C \cap B) = 15/100 = 0.15$ Using the General Addition Rule:

   $$P(C \cup B) = P(C) + P(B) - P(C \cap B)$$

   $$P(C \cup B) = 0.40 + 0.25 - 0.15 = 0.50$$

   The probability is 50%.

   (b) has either cheese or bread on their plates, but not both. This asks for the probability of the symmetric difference, which is the union minus the intersection.

   $$P(C \Delta B) = P(C \cup B) - P(C \cap B)$$

   From part (a), $P(C \cup B) = 0.50$.

   $$P(C \Delta B) = 0.50 - 0.15 = 0.35$$

   Alternatively, this is the probability of having only cheese plus the probability of having only bread: $P(\text{Only } C) = P(C) - P(C \cap B) = 0.40 - 0.15 = 0.25$ $P(\text{Only } B) = P(B) - P(C \cap B) = 0.25 - 0.15 = 0.10$

   $$P(C \Delta B) = 0.25 + 0.10 = 0.35$$

   The probability is 35%.

   (c) has bread on their plates given that they also have cheese on their plates. We need to find the conditional probability $P(B|C)$.

   $$P(B|C) = \frac{P(B \cap C)}{P(C)}$$

   $$P(B|C) = \frac{15/100}{40/100} = \frac{15}{40} = \frac{3}{8} = 0.375$$

   The probability is 37.5%.

   (d) does not have cheese on their plates, if they have bread on their plates. We need to find $P(C'|B)$, where C' is the event "does not have cheese". Using the complement rule for conditional probability:

   $$P(C'|B) = 1 - P(C|B)$$

   First, find $P(C|B)$:

   $$P(C|B) = \frac{P(C \cap B)}{P(B)} = \frac{15/100}{25/100} = \frac{15}{25} = \frac{3}{5} = 0.6$$

   Now, calculate $P(C'|B)$:

   $$P(C'|B) = 1 - 0.6 = 0.4$$

   The probability is 40%.

2. **Don't Take Away Our Drinks**

   Last year, Belgian MPs were told that they would no longer be served free beer during parliamentary sessions. The decision came after an ethics committee report found that a) most workplaces don't serve beer for free, and b) readily available alcohol was making some MPs "quite unpleasant" to work with[1].

   The table showing how each MP voted on the motion to stop serving free beer at work based on their political affiliation.

	In Favour (F)	Against (A)	**Total**
Government (G)	8	75	83
Opposition (O)	7	59	66
Total	15	134	**149**

[1] https://www.politico.eu/article/belgian-mps-to-pay-for-alcohol-parliament-beer-wine/

Assuming that each MP could vote either in favour of the motion or against it, what is the probability that a randomly selected MP

(a) is from the Opposition? From the totals table, the total number of MPs is 149, and the total number of Opposition MPs is 66.

$$P(O) = \frac{\text{Total Opposition}}{\text{Total MPs}} = \frac{66}{149} \approx 0.4430$$

(b) is from the Opposition and voted against the motion? From the table, the number of MPs who are from the Opposition and voted against is 59.

$$P(O \cap A) = \frac{\text{Number}(O \cap A)}{\text{Total MPs}} = \frac{59}{149} \approx 0.3960$$

(c) is in favour of the motion? From the totals table, the total number of MPs who voted in favour is 15.

$$P(F) = \frac{\text{Total Favour}}{\text{Total MPs}} = \frac{15}{149} \approx 0.1007$$

(d) is in favour of the motion if they are from the Opposition? This is a conditional probability, $P(F|O)$. The sample space is restricted to the Opposition MPs (66). Of these 66 MPs, 7 voted in favour.

$$P(F|O) = \frac{\text{Number}(F \cap O)}{\text{Number}(O)} = \frac{7}{66} \approx 0.1061$$

(e) is from the Government given that they voted in favour of the motion? This is a conditional probability, $P(G|F)$. The sample space is restricted to the MPs who voted in favour (15). Of these 15 MPs, 8 are from the Government.

$$P(G|F) = \frac{\text{Number}(G \cap F)}{\text{Number}(F)} = \frac{8}{15} \approx 0.5333$$

3. **Cats and Dogs**

A recent poll in the UK found that 36% of dog owners describe themselves as being "very happy" compared to only 18% of cat owners.

In a group of 1 000 people, 400 own a cat, 250 own a dog, and 150 own a cat and a dog. Determine the probability that a person selected from this group

(a) owns a cat or a dog Let C be the event a person owns a cat and D be the event a person owns a dog. We are given: $N = 1000$ Number(C) $= 400 \implies P(C) = 400/1000 = 0.40$ Number(D) $= 250 \implies P(D) = 250/1000 = 0.25$ Number$(C \cap D) = 150 \implies P(C \cap D) = 150/1000 = 0.15$ Using the General Addition Rule:

$$P(C \cup D) = P(C) + P(D) - P(C \cap D)$$

$$P(C \cup D) = 0.40 + 0.25 - 0.15 = 0.50$$

The probability is 50%.

(b) owns a cat or a dog but not both This asks for the probability of the symmetric difference.

$$P(C \Delta D) = P(C \cup D) - P(C \cap D)$$

From part (a), $P(C \cup D) = 0.50$.

$$P(C \Delta D) = 0.50 - 0.15 = 0.35$$

Alternatively: $P(\text{Only } C) = P(C) - P(C \cap D) = 0.40 - 0.15 = 0.25$ $P(\text{Only } D) = P(D) - P(C \cap D) = 0.25 - 0.15 = 0.10$

$$P(C \Delta D) = 0.25 + 0.10 = 0.35$$

The probability is 35%.

(c) owns a dog, if they own a cat We need to find the conditional probability $P(D|C)$.

$$P(D|C) = \frac{P(D \cap C)}{P(C)}$$

$$P(D|C) = \frac{150/1000}{400/1000} = \frac{150}{400} = \frac{15}{40} = \frac{3}{8} = 0.375$$

The probability is 37.5%.

(d) does not own a cat given that they own a dog. We need to find $P(C'|D)$, where C' is the event "does not own a cat". Using the complement rule for conditional probability:

$$P(C'|D) = 1 - P(C|D)$$

First, find $P(C|D)$:

$$P(C|D) = \frac{P(C \cap D)}{P(D)} = \frac{150/1000}{250/1000} = \frac{150}{250} = \frac{15}{25} = \frac{3}{5} = 0.6$$

Now, calculate $P(C'|D)$:

$$P(C'|D) = 1 - 0.6 = 0.4$$

The probability is 40%.