L16. Hypothesis Test on the Mean; Single Sample, Variance Known

Example 1

Consider a hypothesis test where $H_0 : \mu = 29$ and $H_1 : \mu \neq 29$. A random sample of 25 observations taken from a population produced a sample mean of 25.3. The population is normally distributed with $\sigma = 8$. At the 5% level of significance, is there enough evidence to reject the null hypothesis?

Solution

Example 2

Consider a hypothesis test where $H_0 : \mu = 30$ and $H_1 : \mu < 30$. A random sample of 36 observations taken from a population produced a sample mean of 27.6. The population has a standard deviation of $\sigma = 10$. At the 1% level of significance, is there enough evidence to reject the null hypothesis?

Solution

Example 3

Consider a hypothesis test where $H_0 : \mu = 54$ and $H_1 : \mu > 54$. A random sample of 40 observations taken from a population produced a sample mean of 56.78. The population has a standard deviation of $\sigma = 5.25$. At the 5% level of significance, is there enough evidence to reject the null hypothesis?

Solution

Example 4

A certain colleague of mine who teaches Differential Equations suspects that the 10 ounce bag of fancy Swiss cheese he gets at the supermarket actually weighs less than 10 ounces. He took a random sample of 20 such packages and found that the mean weight for the sample was 9.955 ounces. The population follows a normal distribution with a standard deviation of 0.15 ounces.

At the $\alpha = 0.01$ level of significance does the data indicate that the average weight in this type of packaged cheese weighs less than 10 ounces? Compute a $P - value$ for your test and write a conclusion in the context of the problem.

Solution

Example 5

A study claims that senior citizens living in Mirabel spend an average of 14 hours gardening during the weekend. A random sample of 200 people showed that these senior citizens spend an average of 14.65 hours on gardening during the weekend. Suppose that the standard deviation is known to be 3 hours.

At the 0.025 level of significance does the data indicate that the average amount of time spent on gardening by seniors living in Mirabel is more than 14 hours during the weekend? Compute a $P-value$ and write a conclusion in the context of the problem.

Solution

Example 6

Humans are known to have a mean gestation period of 280 days with a standard deviation of about nine days. A hospital wondered whether there was any evidence that their patients were at risk for giving birth prematurely. In a random sample of 70 women, the average gestation time was 274.3 days. At the 0.005 level of significance does the data indicate that the average mean gestation period for women at this hospital is less than 280 days?

Solution

Example 7

The life in hours of a battery is known to be normally distributed with standard deviation $\sigma = 1.25$ hours. A random sample of 10 batteries has a mean life of $\overline{x} = 40.5$ hours.

a. Is there evidence to support the claim that battery life exceeds 40 hours? Use $\alpha = 0.05$. What is the $P - value$ for this test?

b. Construct an appropriate interval test to support the conclusion obtained in (a)

Solution

Example 8

An engineer who is studying the tensile strength of a steel alloy intended for use in golf club shafts knows that tensile strength is approximately normally distributed with $\sigma = 60$ psi. A random sample of 12 specimens has a mean tensile strength of $\overline{x} = 3450$ psi.

a. Test the hypothesis that mean strength is less than 3500 psi. Use $\alpha = 0.01$.

b. What is the smallest level of significance at which you would be willing to reject the null hypothesis?

c. Construct and explain how you could answer the question in part (a) with a one-sided confidence interval on mean tensile strength.

Solution

Example 9

At a dairy farm, a machine is set to fill 32-ounce cartons with milk. However, the machine does not put exactly 32-ounces into each carton; the amount varies from carton to carton but the volume is known to be normally distributed. When the machine is working correctly, the mean volume dispensed into each carton is 32 ounces, with a standard deviation of 1.5 ounces. A quality control inspector takes 25 cartons can finds that the average volume of milk in the containers is 31.93 ounces.

a. At the 5% level of significance, does the data indicate that the average amount of milk dispensed into the cartons by the machine is different from 32 ounces?

b. Construct and explain how a confidence interval can be used to support the conclusion obtained in (a).

Solution