L17. Hypothesis Test on the Mean; Single Sample, Variance Unknown

Example 1

Consider a hypothesis test where $H_0 : \mu = 205$ and $H_1 : \mu > 205$. A random sample of 14 observations taken from a population that is normally distributed produced a sample mean of 212.37 and a standard deviation of 16.35. At the 1% level of significance, is there enough evidence to reject the null hypothesis?

Solution

Example 2

Consider a hypothesis test where $H_0 : \mu = 50$ and $H_1 : \mu < 50$. A random sample of 8 observations taken from a population that is normally distributed produced a sample mean of 44.98 and a standard deviation of 6.77. At the 5% level of significance, is there enough evidence to reject the null hypothesis?

Solution

Example 3

Consider a hypothesis test where $H_0 : \mu = 10.70$ and $H_1 : \mu \neq 10.70$. A random sample of 47 observations taken from a population produced a sample mean of 12.025 and a standard deviation of 4.90. At the 1% level of significance, is there enough evidence to reject the null hypothesis?

Solution

Example 4

The President of a university claims that the mean time spent partying by students at this university is less 11 hours per week. A random sample of 40 students taken from this university showed that they spent an average of 10.5 hours partying, with a standard deviation of 2.3 hours. Test the President's claim at the 2.5% level of significance.

Solution

Example 5

Before Listerine was sold as a mouthwash, it was used as a surgeon's antispetic (in its undiluted form), advertised as a floor cleaner, and even touted as a cure for gonorrhea.

Listerine mouthwash is sold in 1.5 litre bottles. The machine responsible for filling the bottles is checked for regularly to ensure that it is properly calibrated. A random sample of 32 bottles were taken from a production run, and it was found that the average amount of mouthwash in the bottles was 1.507 litres, with a standard deviation of 0.03 litres.

At the 10% level of significance, does the data indicate that the machine is dispensing a volume that is different from 1.50 litres, and hence requires recalibration? What is the $P-$value for this test?

Solution

Example 6

The mean balance of all checking accounts at a bank on December 31, 2011 was \$850. A random sample of 55 checking accounts taken recently from this bank gave a mean balance of \$880 with a standard deviation of \$75. Using a 1% significance level, can you conclude that the mean balance has increased during this period? What is the $P-value$ for this test?

Solution

Example 7

The article *A Critical Appraisal of 98.6 Degrees F, the Upper Limit of the Normal Body Temperature, and Other Legacies of Carl Reinhold August Wunderlich* reported body temperature, gender, and heart rate for a number of subjects. The body temperatures for 25 female subjects resulted in a sample average of $\overline{x} = 98.264°F$ and a standard deviation of $s = 0.4821°F$.

a. Assuming that the population is normally distributed, does the data indicate at the 0.05 level of significance that the average body temperature for women is different from $98.6°F$ What is the $P - value$?

b. Construct and explain how the question in (a) could be answered by constructing a two-sided confidence interval on the mean female body temperature.

c. State the Type I and Type II error in the context of the problem.

Solution

Example 8

A manufacturer of running shoes knows that the average lifetime for a particular model of shoes is 15 months. Someone in the research and development division of the shoe company claims to have developed a longer lasting product. This new product was worn by 36 individuals and lasted on average for 17 months. The variability of the original shoe is estimated based on the standard deviation of the new group which is 5.5 months.

a. At the 2.5% level of significance, test the designer's claim that he has developed a shoe which lasts longer than 15 months and write a conclusion in the context of the problem. What is the $P-value$ for this test?

b. Construct an appropriate one-sided confidence bound to support the conclusion obtained in (a)

c. State the Type I and Type II errors in the context of problem.

Solution

Example 9

Sweethearts, or conversation hearts, are small heart shaped candies sold around Valentine's Day. First produced in 1901, the candies feature classic messages like: I ♡ You, Let's Kiss, and True Love. NECCO, the company which makes conversation hearts, adds 10 new messages each year. Recent additions include: Text Me, Hey You!, and Yeah Right.

The company claims that conversational hearts have a shelf life of 5 years. A random sample of 200 hearts taken from the warehouse found that the average shelf life of the sample was 58 months with a standard deviation of 4.5 months. Assume that the population is normally distributed.

a. At the 0.05 level of significance, does the data indicate that the average shelf life of these candies is less than 5 years? What is the $P-value$ for this test.

b. Construct and explain how an appropriate interval estimate could be used to support the conclusion obtained in (a).

Solution