Biometry - HW2 - Solutions

(1) $p(x) = {}_nC_x\, p^x (1-p)^{n-x} = {}_5C_x\, (0.37)^x (0.63)^{5-x}$, $n=5, p=0.37$

x	0	1	2	3	4	5
$p(x)$	0.099	0.291	0.342	0.201	0.059	0.007

(2) $p(x) = \dfrac{e^{-\mu}\mu^x}{x!} = \dfrac{e^{-1.6}(1.6)^x}{x!}$, $\mu = 1.6$

x	0	1	2	≥ 3
$p(x)$	0.202	0.323	0.258	0.217

(3) $E(X) = \dfrac{(1-p)r}{p} = \dfrac{(1-0.22)\cdot 4}{0.22} = 14.2$; $\sigma = \sqrt{\dfrac{(1-p)r}{p^2}} = \sqrt{\dfrac{(1-0.22)4}{(0.22)^2}} = 8.0$

$p(x) = {}_{x+r-1}C_{r-1}\, p^r (1-p)^x$, $p(10) = {}_{13}C_3\, (0.22)^4 (0.78)^{10} = 0.0558$

(4)

x	≤ 1	2	3	4
Observed freq.	4	10	38	48
$p(x)$	0.018	0.125	0.393	0.463
Expected freq.	1.77	12.51	39.31	46.33

$\bar{x} = \dfrac{(1)(4)+(2)(10)+(3)(38)+(4)(48)}{100} = 3.3$

$s^2 = \dfrac{1}{99}\left[(1-3.3)^2 \times 4 + \ldots + (4-3.3)^2 \times 48\right] = 0.6566$

$CD = \dfrac{s^2}{\bar{x}} = \dfrac{0.6566}{3.3} = 0.20$; binomial seems appropriate

$p = \dfrac{\bar{x}}{n} = \dfrac{3.3}{4} = 0.825$

$\chi^2 = \dfrac{(4-1.77)^2}{1.77} + \dfrac{(10-12.51)^2}{12.51} + \dfrac{(38-39.31)^2}{39.31} + \dfrac{(48-46.33)^2}{46.33} = 3.42$

Degrees of freedom : $4-1-1=2$; $0.10 < p\text{-value} < 0.50$ Decent fit.

$n=3, p=0.825$; $p(0) = {}_3C_0\, (0.825)^0 (0.175)^3 = 0.0054$. Very unlikely.

(5)

x	0	1	2	3	4
Observed freq.	211	128	36	20	5
$p(x)$	0.497	0.348	0.122	0.028	0.005
Expected freq.	198.6	139.0	48.7	11.4	2.0

$\bar{x} = 0.7$, $s^2 = 0.84$, $CD = \dfrac{0.84}{0.7} = 1.2$

Poisson distribution doesn't look a good fit already.

$\mu \approx \bar{x} = 0.7$

$\chi^2 = \dfrac{(198.6-211)^2}{198.6} + \ldots + \dfrac{(2.0-5)^2}{2.0} = 16.1$ Degrees of freedom $= 5-1-1 = 3$

$p\text{-value} < 0.005$ The Poisson distribution does not fit this data. Since $CD > 1$ The negative binomial distribution might be a good fit.

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(6) $\bar{x} = \dfrac{2+9+0+10+8+0+1}{7} = 4.29$

$$s^2 = \frac{1}{6}\left[(4.29-2)^2 + (4.29-9)^2 + (4.29-0)^2 + (4.29-10)^2 + (4.29-8)^2 + (4.29-0)^2 + (4.29-1)^2\right] = 20.24$$

$$CD = \frac{20.24}{4.29} = 4.72 > 1$$

During the dry season all animals converge on the available waterholes.

(7) Expected frequencies under independence are in the table on the right.

	Blue	Gold	Red	
Dead	7	9	24	40
Alive	129	46	215	390
	136	55	239	430

	Blue	Gold	Red	
Dead	12.7	5.1	22.2	40
Alive	123.3	49.9	216.8	390
	136	55	239	430

$$\chi^2 = \frac{(7-12.7)^2}{12.7} + \dots + \frac{(215-216.8)^2}{216.8} = 6.27$$

Degrees of freedom $(2-1)(3-1) = 2$

p-value $= 0.0435 < 0.05$

Fatality rate and uniform colour are dependent.