BIOMETRY, HOMEWORK 3

(1) (Normal distribution $X \to z \to p$) The historical March temperature average for the town of Inuvik, NWT is $-16.8C^\circ$ with standard deviation of $4.4C^\circ$. It was widely reported reported that in 2019 the March avetrage temperature in Inuvik was $-4.0C^\circ$. Assuming that the distribution of March temperature averages in Inuvik is normal compute the probability that according to the historical model the the March average will be $-4.0C^\circ$ or more. Is this event statistically significant. If yes, could you offer an explanation for the 'historically' ulikely event?

(2) (Normal distribution $p \to z \to X$) Wolverines in Siberia have low population density and require a very large home range. The ranges of female wolverines are normally distributed with an average of $190km^2$ and standard deviation of $35km^2$. Determine the home ranges of the female Siberian wolverines that sit at the bottom 5% and the top 5% of the distribution.

(3) (Confidence interval for the mean, large sample)

$$\bar{x} - z_{\alpha/2}\frac{s}{\sqrt{n}} \leq \mu \leq \bar{x} + z_{\alpha/2}\frac{s}{\sqrt{n}}$$

A random sample of 61 adult secretary birds had mean weight of $4.02kg$ with standard deviation of $0.61kg$. Construct a 95% and a 99% confidence intervals for the population average weight of secretary birds. Write a sentence commenting on the tension between confidence and precision when using confidence interval esimates.

(4) (Confidence interval for the mean, small sample)

$$\bar{x} - t_{\alpha/2}\frac{s}{\sqrt{n}} \leq \mu \leq \bar{x} + t_{\alpha/2}\frac{s}{\sqrt{n}}$$

The average high May temperature, in C° at the Dorval Airport for the last seven years is given below

17.5 18.0 19.0 18.8 19.5 19.8 18.3

Assuming the average high May temperature at Dorval is normally distributed, compute a 99% confidence interval for the population mean.

(5) (Confidence interval for the difference of means, large samples)

$$\bar{x}_1 - \bar{x}_2 - z_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \leq \mu_1 - \mu_2 \leq \bar{x}_1 - \bar{x}_2 + z_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

An article in *Comm. Soil Science and Plant Analysis*, 2001, describes chemical analysis of soil taken from a farm in Western Australia. Fifty specimens were taken at depths 50 and 250 cm. At a depth of $50cm$, the average NO_3 concentration (in mg/L) was 88.5 with a standard deviation of 49.4. At a depth of $250cm$, the average concentration was 110.6 with a standard deviation of 51.5. Determine a 95% confidence interval for the difference between the NO_3 concentrations at the two depths.

(6) (Confidence interval for the difference of means, small samples)

$$\bar{x}_1 - \bar{x}_2 \pm t_{\alpha/2}\sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}}\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

Good website design can make Web navigation easier. An article in *Int. J. of Human-Computer Interaction*, 2001, presents a comparison of item recognition between two designs. A sample of 10 users using a conventional Web design averaged 32.3 items identified, with standard deviation of 8.56. A sample of 10 users using a new structured Web design averaged 44.1 items identified, with a standard deviation of 10.09. Compute a 95% confidence interval for the difference of items identified between the two designs.

(7) (Confidence interval for the difference of mean, paired samples)

$$\bar{d} - t_{\alpha/2}\frac{s_d}{\sqrt{n}} \leq \mu_d \leq \bar{d} + t_{\alpha/2}\frac{s_d}{\sqrt{n}}$$

The Wisconsin DNR did a study on the effects of motorized watercraft on aquatic ecosystems. Two stroke motors can emit 25-30% of their unburned gas and oil mixture into the water. A two year study of 6 lakes showed the following oil product concentration in surface water (ppm)

Location	1	2	3	4	5	6
2017	272	181	272	197	245	222
2018	251	193	268	201	276	298

Construct a 95% confidence interval for the difference of oil product concentrations between 2017 and 2018. Based on this analysis is the water in

these 6 lakes more polluted in 2018?

(8) (Confidence interval for proportion)

$$\hat{p} - z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \leq p \leq \hat{p} + z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

In a sample of 2314 randomly selected American elm trees in Eastern Canada 458 were affected by Dutch elm disease. Compute a 90% confidence interval for the proportion of American elm trees in Eastern Canada affected by this disease.