BIOMETRY, HOMEWORK 4

(1) (Hypothesis testing for the mean, large sample)

$$z = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}}$$

Humans are known to have a mean gestation period of 280 days (from last menstruation) with standard deviation of 9 days. A hospital wondered whether there was any evidence that the patients at their maternity ward were at risk for giving birth prematurely. In a random sample of 70 women, the average gestation time was 274.3 days. At the 0.05 level of significance test $H_0 : \mu = 280$ against $H_1 : \mu < 280$. Report a p-value and make sure to draw a conlusion in the context of the problem.

(2) (Hypothesis testing for the mean, small sample)

$$t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}}$$

An Internet database claims that snowy owls live on average 9.4 years in the wild. A sample of 6 snowy owls tracked by your team over the years had the following lifespans

5.6 8.0 3.7 6.9 9.6 5.6

Use this data to test the null hypothesis $H_0 : \mu = 9.4$ versus the alternative $H_1 : \mu < 9.4$ at the 5% level of significance. Make sure to report a p-value and draw a conclusion in the context of the problem.

(3) (Hypothesis testing for the difference of means, paired samples)

$$t = \frac{\bar{d} - \mu_d}{\frac{s}{\sqrt{n}}}$$

The latency time for the reaction of nerve fibres in the arm was measured before and after applying an ice pack to the area at six locations. The results (ms) are given in the table bellow.

Location	1	2	3	4	5	6
Before ice is applied	27.2	18.1	27.2	19.7	24.5	22.2
After ice is applied	25.1	19.3	26.8	20.1	27.6	29.8

Use this data to test $H_0 : d = 0$ against $H_1 : d > 0$ at the 5% level of significance. Report a p-value for the test. Here, $d = \mu_2 - \mu_1$. Draw a conclusion in the context of the problem.

(4) (Confidence interval for the variance (st. dev.) of a normal popualtion)

$$\frac{(n-1)s^2}{\chi^2_{\alpha/2}} \leq \sigma^2 \leq \frac{(n-1)s^2}{\chi^2_{1-\alpha/2}}$$

The average high May temperature, in C° at the Mirabel Airport for the last seven years is given below

17.5 18.0 19.0 18.8 19.6 20.8 20.3

Assuming the average high May temperature at Mirabel is normally distributed, compute a 95% confidence interval for the population standard deviation.

(5) (Hypothesis testing for the standard deviation of normal)

$$\chi^2 = \frac{(n-1)s^2}{\sigma_0^2}, \quad n-1 \text{ d.f.}$$

A manufacturer of solar panels claims that the mean life span of their panels is 12 years with a standard deviation of 9 months. If a random sample of 10 of these panels has a standad deviation of 15 mothhs do you think that the standard deviation is more than 9 months. Assume the life-span is normally distributed and use $\alpha = 0.05$-level of significance.

(6) (Two sample testing for difference of means; variances presumed equal)

$$s_p^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}$$

$$t = \frac{\bar{X}_1 - \bar{X}_2 - \Delta}{s_p\sqrt{1/n_1 + 1/n_2}}, \quad n_1+n_2-2 \text{ d.f.}$$

In April two years ago, chemical analysis were made of 24 water samples taken from various parts of a city lake, and the measurements of chlorine content were recorded. During the next two winters, the use of road salt was substantially reduced in the catchment areas of the lake. This April, 27 water saples were analyzed and their chlorine content recorded. Here are the averages and the st.dev from two years ago: $X_1 = 18.3, s_1 = 1.2$ and this year: $X_2 = 17.5, s_2 = 1.1$. Assuming the populations are normal with equal variances test $H_0 : \mu_1 = \mu_2$ versus $H_1 : \mu > \mu_2$ at the $\alpha = 0.05$ level of significance. Report a p-value and draw a conclusion in the context of the problem.

(7) (Two sample testing for difference of means; unequal variances)

$$\nu = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{(s_1^2/n_1)^2}{n_1-1} + \frac{(s_2^2/n_2)^2}{n_2-1}}$$

$$t = \frac{\bar{X}_1 - \bar{X}_2 - \Delta}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \quad \nu \text{ d.f.}$$

A new method of storing snap beans is believed to retain more ascorbic acid than an old method. In an experiment, snap beans were harvested under uniform conditions and frozen in 18 equal-size packages. Nine of these packages were randomly selected and stored according to the new method, and the other 9 packages were stored by the old method. Subsequently ascorbic acid deteminations were made (mg/kg): $x_{old} = 410, s_{old} = 45$; $x_{new} = 454, s_{new} = 18$. Do these data substantiate the claim that more ascorbic acid is retained under the new method of storing? Assuming the populations are normal implement a statistical test at the $\alpha = 0.05$ level of significance.