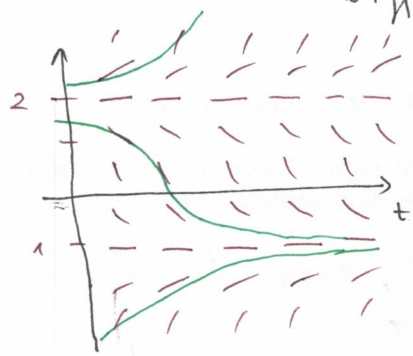
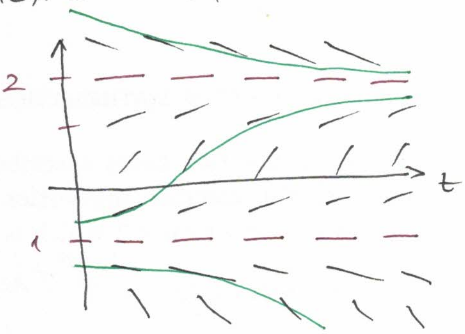
Diff Eq - Clex 1 - Solutions

(1)





All derivatives, and hence all slopes changed by a factor of -1.

(2) $4\frac{dy}{dt} = 2-y$, $\frac{dy}{2-y} = \frac{dt}{4}$, $\int \frac{dy}{2-y} = \int \frac{dt}{4}$, $-\ln|2-y| = \frac{t}{4} + C_1$

$2-y = \pm e^{-t/4-C}$, $y = 2+ke^{-t/4}$, $-1 = y(0) = 2+k$, $k=-3$; $\boxed{y(t) = 2-3e^{-t/4}}$

(3) $y' + 2y = 4-6t$, Integrating factor $\mu(t) = e^{2t}$

$\frac{d}{dt}[e^{2t}y] = e^{2t}(4-6t) \Rightarrow e^{2t}y = \int 4e^{2t}\,dt - \int 6te^{2t}\,dt$

$e^{2t}y = 4\,e^{2t}/2 - 6t\,e^{2t}/2 + \int 6\,e^{2t}/2\,dt = 2e^{2t} - 3te^{2t} + \frac{3}{2}e^{2t} + C$

$y(t) = e^{-2t}\left[\frac{7}{2}e^{2t} - 3te^{2t} + c\right] = \frac{7}{2} - 3t + Ce^{-2t}$

$\lim_{t\to\infty} y(t) = -\infty$ for all values of C.

(4) $y' - \frac{4y}{t} = \frac{1}{2}t^5e^{-t}$, $\mu(t) = \exp\left\{\int -\frac{4}{t}\,dt\right\} = ct^{-4}$

$\frac{d}{dt}[t^{-4}y] = \frac{1}{2}t^5e^{-t}\cdot t^{-4} = \frac{1}{2}te^{-t}$

$t^{-4}y = \frac{1}{2}\int te^{-t}\,dt = -\frac{1}{2}te^{-t} + \frac{1}{2}\int e^{-t}\,dt = -\frac{1}{2}te^{-t} - \frac{1}{2}e^{-t} + C$

$y(t) = -\frac{1}{2}t^5e^{-t} - \frac{1}{2}t^4e^{-t} + Ct^4$

$0 = y(1) = -1/2\,e^{-1} - 1/2\,e^{-1} + C \Rightarrow C = e^{-1}$

$y(t) = -1/2\,t^5e^{-t} - 1/2\,t^4e^{-t} + e^{-1}t^4$

$\lim_{t\to\infty} y(t) = -1/2\underbrace{\lim_{t\to\infty} t^5e^{-t}}_{\to 0} - 1/2\underbrace{\lim_{t\to\infty} t^4e^{-t}}_{\to 0} + e^{-1}\lim_{t\to\infty} t^4 = +\infty$