DIFFERENTIAL EQUATIONS, CLASS EXERCISE 1

(1) Draw the directional fields for the two differential equations $y' = (y+1)(y-2)$ and $y' = (y+1)(2-y)$. Make sure to include the equilibrium solutions. Write a sentence explaining what changed from the first equation to the second.

(2) Solve the initial value problem

$$4y' = 2 - y, \quad y(0) = -1.$$

(3) Find the general solution of the differential equation

$$\frac{dy}{dt} + 2y = 4 - 6t.$$

The general solution depends on a parameter. For what values of this parameter the solution tends to $+\infty$ and for what values to $-\infty$ as $t \to \infty$?

(4) (i) Solve the initial value problem

$$2\frac{dy}{dt} - \frac{8y}{t} = t^5 e^{-t}, \quad y(1) = 0$$

(ii) What is the limiting behaviour of the solution as $t \to \infty$?