

DIFFERENTIAL EQUATIONS, CLASS EXERCISE 11

- (1) Use Laplace transform to solve the initial value problem.

$$y'' - 10y' + 9y = 5t, \quad y(0) = -1, \quad y'(0) = 2.$$

- (2) Use Laplace transform to solve the initial value problem.

$$y'' - 6y' + 15y = 2 \sin 3t, \quad y(0) = -1, \quad y'(0) = -4.$$

- (3) Find the Laplace transform of $f(t)$:

$$f(t) = \begin{cases} 0 & t < 1, \\ t^2 - 2t + 2 & t \geq 1. \end{cases}$$

- (4) Find the inverse Laplace transform

$$\mathcal{L}^{-1} \left\{ \frac{e^{-2s}}{s^2 + s - 2} \right\}.$$

- (5) Use Laplace transform to solve the initial value problem.

$$y'' + 4y = g(t), \quad y(0) = 1, \quad y'(0) = 0,$$

where

$$g(t) = \begin{cases} 0 & 0 \leq t < \pi, \\ 1 & \pi \leq t < 2\pi, \\ 0 & 2\pi \leq t. \end{cases}$$