DIFFERENTIAL EQUATIONS, CLASS EXERCISE 2

(1) Solve the DE

$$y' - \frac{y}{3} = \sin t$$

Above the t-interval $[0, 3\pi]$ sketch the integral curves for the five initial conditions

$$y(0) = -1.3, \quad y(0) = -1.1, \quad y(0) = -0.9, \quad y(0) = -0.7, \quad y(0) = -0.5$$

on the same graph.

Do you see how the integral curves are diverging from one 'stable' solution? What is the formula of this 'stable' solution?

(2) (i) Solve the initial value problem

$$y' = xy^3(1 + x^2)^{-1/2}, \quad y(0) = 1.$$

(ii) Determine the interval on which the solution is defined.

(3) A second order chemical reaction involves the interaction of one molecule of substance P with one molecule of substance Q to produce one molecule of a new substance X; this is denoted by $P + Q \to X$. Suppose that p and q are the initial abundances (in moles) of P and Q respectively. Let $x(t)$ be the amount of X produced by the reaction at time t (in minutes). The rate at which the reaction occurs is given by the equation

$$\frac{dx}{dt} = \alpha(p - x)(q - x),$$

where α is a positive constant.

Assuming that $p = 10$, $q = 5$ and $\alpha = 0.4$ determine the amount of X as a function of t, i.e. solve the DE for the function $x(t)$. How long it will take for all of Q to be completely consumed by the reaction?

(4) Solve the initial value problem

$$y' + y^3 = 0, \quad y(0) = y_0.$$

Next, descibe the interval in which the solution exists in terms of the initial value y_0.

(5) An island nation in the Caribbean has a population of one hundred thousand. A businessman has just returned to the island from a trip to South America and is infected with the Xico virus. Immediately, the mosquitoes on the island have started spreading the Xico virus throughout the human population. Assume the number of infected islanders follows the logistic model

$$\frac{dN}{dt} = 0.3\left(1 - \frac{N}{100000}\right)N,$$

where the time is in days and $N(0) = 1$.

i) Solve the initial value problem without using the general formula derived in class.

ii) How long it will take for 95% of the island population to become indected?