Diff Eq - Clex 4 - Solutions

①

n	t_n	y_n	y_n'	exact
0	0	0	1	0
1	0.2	0.2	1.019	0.201
2	0.4	0.404	1.074	0.411
3	0.6	0.619	1.167	0.637
4	0.8	0.852	1.302	0.888
5	1	1.112	.	1.175

$\frac{dy}{dt} = y + e^{-t}$. $\mu(t) = e^{-t}$

$y(t) = -\frac{1}{2}e^{-t} + ce^{t}$

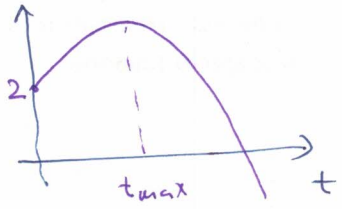
$y(0) = 0 \Rightarrow c = 1/2$

$y(t) = -\frac{1}{2}e^{-t} + \frac{1}{2}e^{t}$

The exact solution contains a growing exponent and the approximation is less and less accurate with increasing t.

② $2r^2 - 3r + 1 = 0$; $r_1 = 1$, $r_2 = 1/2$

$\Rightarrow y(t) = c_1 e^{t} + c_2 e^{t/2}$



$2 = c_1 + c_2$; $1/2 = c_1 + c_2/2 \Rightarrow c_1 = -1$, $c_2 = 3$

$y(t) = -e^{t} + 3e^{t/2}$, $\lim_{t\to\infty} y(t) = -\infty$

$y'(t) = -e^{t} + \frac{3}{2}e^{t/2} = 0 \Rightarrow t_{max} = 2\ln\frac{3}{2}$, $y_{max} = 9/4$

③ $y_1'' - 2y_1' + y_1 = e^{t} - 2e^{t} + e^{t} = 0$. $y_2'' - 2y_2' + y_2 = (2e^{t} + te^{t}) - 2(e^{t} + te^{t}) + te^{t} = 0$

$$W(y_1, y_2) = \begin{vmatrix} e^{t} & te^{t} \\ e^{t} & e^{t} + te^{t} \end{vmatrix} = e^{2t} \neq 0$$

$\Rightarrow \{y_1, y_2\}$ - form a fundamental set.

④

$$\begin{vmatrix} f_1 & f_2 & f_3 \\ f_1' & f_2' & f_3' \\ f_1'' & f_2'' & f_3'' \end{vmatrix} = \begin{vmatrix} 1 & \sin^2 x & \cos^2 x \\ 0 & 2\sin x\cos x & -2\sin x\cos x \\ 0 & 2(\cos^2 x - \sin^2 x) & -2(\cos^2 x - \sin^2 x) \end{vmatrix} = 0$$